Chapter 17 non calculus

Electric Charge and Coulomb's Law

When he flew a kite in a thunderstorm, Ben Franklin both avoided killing himself and demonstrated that lightning was an electrical phenomenon. (The Russian scientist Georg-Wilheim Richman repeated Franklin's experiment and was killed in the process.) Franklin also introduced the concept of positive and negative charge to describe the two kinds of charge observed in electrostatic experiments.

In this chapter we will start with some simple electrostatic experiments that illustrate the fact that there are two kinds of electric charge. The experiments will also demonstrate why it is difficult to determine the electric force law from such eighteenth century apparatus.

In the nineteenth century, Charles Coulomb did careful experiments to verify that the electric force was a $1/r^2$ *force law, and that the force between two charged objects is proportional to the product of the charges on the objects. Because of this work the electric force law is known as* **Coulomb's law** *and electric charge is measured in units called* **coulombs**.

In the twentieth century we finally learned what carried electric charge. In the matter around us that we are familiar with, there are four kinds of basic particles. There are the protons and neutrons found in the atomic nucleus, the electrons that surround the nucleus to form complete atoms, and particles of light, called photons, that are the source of visible light. Of these particles, the photon and the neutron are electrically neutral, while the proton carries one kind of charge and the electron carries the other. Following Franklin's convention, the charge on the proton is positive and that on the electron is negative. And the fact that an electrically neutral atom contains equal numbers of protons and electrons tells us that the charge on the proton and electron are equal in magnitude, while opposite in sign.

After our discussion of the electrostatic experiments, we will compare Coulomb's $1/r^2$ *electric force law with Newton's* $1/r^2$ *gravitational force law. While there are many similarities, there are some very important differences. We will then go on to a 20th century point of view of electric forces.*

Two Kinds of Electric Charge

Some simple electrostatic experiments clearly demonstrate that there are two kinds of electric charge. For these experiments we use two small Styrofoam™ balls wrapped with aluminum foil and suspended by threads as shown in Figure (1a). We also need one glass rod, one rubber rod, a piece of silk cloth and a piece of cat fur. (We assume that these latter items were commonly available in eighteenth century physics labs.)

In the first experiment we rub the glass rod with the silk cloth and touch both balls with the glass rod. When things have settled down, we observe that the two balls are repelling each other as shown in Figure (1b).

Some peculiar things happen before we get the result shown in Figure (1b). As we bring the glass rod up to the first ball, the ball is attracted to the glass rod. After we touch the glass rod, the ball is repelled. Before we touch the second ball, we notice that it is attracted to the first ball. Only after we touch it with the glass rod do we see the balls repel each other as shown in Figure (1b).

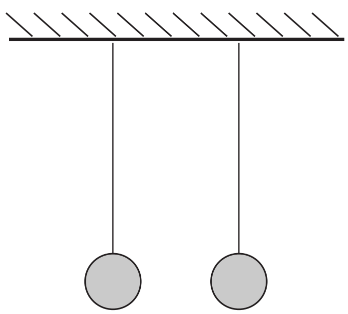


Figure 1a
Styrofoam balls wrapped in aluminum foil and suspended on threads.

For the second experiment, we first touch both balls with our hand, and notice that afterward the balls hang straight down. Our hand has removed any electric charge placed on the balls by the glass rod. Then we rub the rubber rod with cat fur and touch both balls with the rubber rod. When things settle down, we see that the balls again repel each other as shown in Figure (1b).

The third experiment is the crucial one. Again we discharge both balls by touching them, so that they hang straight down as in Figure (1a). Then we touch one ball with the glass rod rubbed by silk, and the other ball with the rubber rod rubbed by cat fur. Instead of repelling each other as in Figure (1b), the two balls attract each other as shown in Figure (1c). This clearly demonstrates that there is something different about the charges deposited by the glass rod and the rubber rod. If the charges were the same kind on the two rods, the balls would still repel each other. Thus there are at least two kinds of electric charge.

A fourth experiment, which is a bit more complex than the first three, tells us more about the nature of electric charge. After you have charged the balls with different rods to get the attractive force shown in Figure (1c), move the threads together so that the balls touch each other. (Be careful that you do not touch the balls.) When you let go of the threads, one of two things happens. Either the balls then repel each other or hang straight down.

Most of the time the balls will repel somewhat. But if you get just the right amount of charge on each ball (by different amounts of rubbing of the rods), you can end up with the balls hanging straight down. When this happens, you can see that the charges on

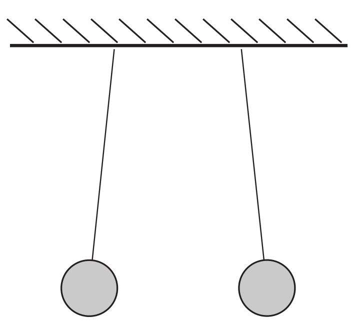


Figure 1b
After the two balls are touched by the same rod.

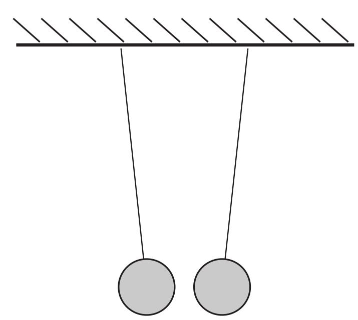


Figure 1c
After the two balls are touched by different rods.

the two balls cancelled each other. Such results led Franklin to name the two kinds of charge ***positive*** and ***negative*** charge. When you got equal amounts of positive and negative charge on the balls, then had them touch, the two charges cancelled each other leaving the balls uncharged.

Not knowing the origin of electric charge, Franklin simply chose to call the charge left on the glass rod positive charge, and that left on the rubber rod negative charge. Jumping to the 20th century, we know that the two kinds of charge are carried in ordinary matter by electrons and protons. Franklin's choice of the glass rod being positive leads to the proton being positive and the electron being negative.

The electrostatic experiments also demonstrate that electric charge is quite mobile. We could get charge on the rods by rubbing them, and we could transfer charge by touching the balls with the rods.

In the twentieth century we learned that this mobility of electric charge is caused by the fact that electrons can flow as a fluid through a conductor or be scraped off the surface of an insulator by rubbing.

When we rub the glass rod with a silk cloth, electrons are rubbed off the rod and stick to the cloth. The lack of electrons leaves more protons than electrons on the glass rod, and the glass rod ends up with a positive charge. When we touch the glass rod to the aluminum foil surrounding the ball, the positive glass rod sucks some electrons out of the aluminum foil leaving the ball positively charged.

On the other hand when we rub the rubber rod with cat fur, electrons are scraped off the cat fur and stick to the rubber rod. When the rubber rod touches the aluminum foil on the ball, electrons pass from the rod to the foil leaving the ball negatively charged.

That still leaves the question of why the neutral ball was attracted to a charged rod before the rod touched the ball. The answer is due to the mobility of the electrons in the aluminum foil. When we brought the positive rod up to the ball, the positive charge on the rod attracted electrons in the foil to the front side of the ball leaving a net positive charge on the back side as indicated in Figure (2a).

Since the negative charge is closer to the positive rod, the attractive force of the negative charge is stronger than the repulsive force of the positive charge, and there is a net force toward the rod. This is true even though there is no net charge on the ball.

We still get an attractive force as the negative rubber rod is brought up to the ball, because the minus charge on the ball pushes the electrons in the foil toward the back side of the ball, leaving a net positive charge on the front side, as indicated in Figure (2b). Since the plus charge on the ball is closer to the negative rod, it is more strongly attracted than the more distant negative charge is repelled.

Exercise 1

Explain why the ball jumps away from the rod after the rod touches the ball.

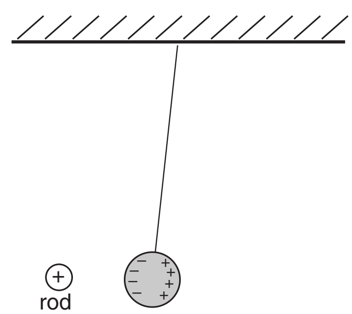


Figure 2a
The + rod attracts electrons to the front side.

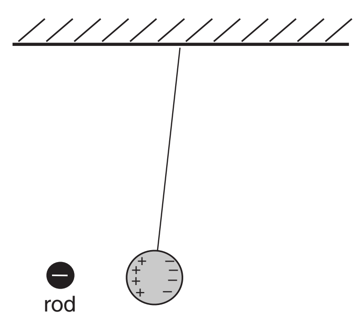


Figure 2b
The – rod pushes the electrons to the back side.

COULOMB'S ELECTRIC FORCE LAW

In the study of gravity, it is not difficult to demonstrate that the gravitational force between objects like stars and planets is accurately described by a $1/r^2$ force law. Newton knew this from his analysis of the Kepler orbits of the planets. We could determine this from our computer analysis of planetary orbits in Chapter 8. We saw that if we made even a small change in the exponent, from $1/r^2$ to $1/r^{1.9}$ we got the big change in the orbit shown in Figure (8-25) repeated here. When the exponent is not exactly $1/r^2$, the ellipse starts to precess.This precession is not present in a pure Kepler orbit.

In contrast, due to the mobility of electric charge, it is very difficult to show experimentally that the electric force law is a $1/r^2$ force. We just saw, for example, that the mobility electrons can even lead to an electric force between a charged rod and an electrically neutral ball. As we mentioned in the introduction, Charles Coulomb did some very careful experiments accurately verifying the $1/r^2$ nature of the electric force between point particles.

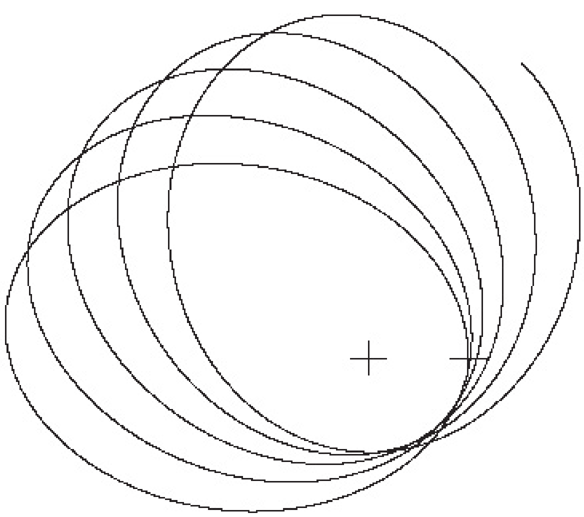


Figure 8-25 (repeated)
Planetary orbit when the gravitational force is modified to a $1/r^{1.9}$ force.

In many ways Coulomb's electric force law can be modeled on Newton's gravitational force law. For gravity the gravitational force F_g between two point masses m_1 and m_2 separated by a distance r is

$$F_g = \frac{Gm_1m_2}{r^2} \qquad \textit{Newton's law of gravity} \qquad (9\text{-}3)$$

where G is the universal gravitational constant. Coulomb's law for electricity can be written in the form

$$F_e = \frac{KQ_1Q_2}{r^2} \qquad \textit{Coulomb's electric force law} \qquad (1)$$

Where Q_1 and Q_2 are two point charges separated by a distance r, and K is the universal electric force constant.

In order to work with Newton's law of gravity, one must first have an experimental definition of mass. We still use, as the definition of a unit mass, the kilogram block of platinum kept in Paris, France. Copies of the unit mass are sent around the world so that various laboratories can calibrate their own sets of standard masses. Once you know what the masses m_1 and m_2 are in terms of the standard unit mass, you can then do a Cavendish experiment, measuring the force F_g when the masses are a distance r apart, and then solve for the gravitational constant

$$G = \frac{r^2F_g}{m_1m_2} \qquad (2)$$

The main point we just tried to make is that the value of the universal gravitational constant G depends on our choice of a unit mass. In the MKS system where mass is in kilograms and distance in meters, G has the value

$$G = 6.67 \times 10^{-11} \frac{\text{newton meter}^2}{\text{kilogram}^2}$$

In the CGS system, where mass is in grams, distance in centimeters and force measured in dynes (1 dyne $= 10^{-5}$newtons) the universal gravitational constant is

$$G = 6.67 \times 10^{-8} \frac{\text{dyne cm}^2}{\text{gm}^2}$$

Imagine that we could use a similar procedure for Coulomb's electric force law. We create a unit charge and store it in a laboratory in Paris. We then send copies of the unit charge around the world so that each laboratory could calibrate their own set of standard charges. Then the laboratory responsible for physical standards (in the U.S. it is *The Institute of Standards and Technology*) would measure the electric force F_e between two known charges Q_1 and Q_2 separated by a distance r. They could then solve for the universal electric constant K, getting

$$K = \frac{r^2 F_e}{Q_1 Q_2} \qquad (3)$$

The only problem with this approach is that you cannot easily store a unit charge. As we saw, if you put a charge on a glass or rubber rod and touch a piece of metal, the charge flows off the rod onto the metal. On a damp day the charge will flow off the rod into the moist atmosphere. In other words the approach used for the gravitational force law, first define the unit mass and use that to determine the universal force constant, does not work for electricity.

Instead, in electrical theory, one first defines the electric force constant K, and then one uses Coulomb's law to determine how big a unit charge is. For example, if you took two identical charges Q, separated them by a distance r, and measured the electric force F_e between them, you would get

$$F_e = \frac{KQ^2}{r^2} \; ; \qquad Q = r\sqrt{\frac{F_e}{K}} \qquad (4)$$

as the value of the charge Q.

MKS and CGS Units

As we saw in the case of gravity, the numerical value of the universal force constant G depends on the system of units used. The difference for electricity is that we first make an arbitrary choice for the universal force constant K, and later determine how big the unit charge is.

In the CGS system, a very simple choice is made, namely

$$K = 1 \qquad \textit{electric force constant in the CGS system} \qquad (5)$$

with the result that Coulomb's law becomes

$$F_e = \frac{Q_1 Q_2}{r^2} \qquad \textit{Coulomb's law in CGS units} \qquad (6)$$

The simplicity of this choice makes the CGS system of units sometimes useful in describing atomic phenomena.

On the other hand the CGS system is nearly a disaster when discussing practical electrical phenomena that we deal with every day. We are all familiar with volts (AA batteries are 1.5 volts), amps (household circuits usually now have 20 amp circuit breakers) and watts (you can buy 100 watt bulbs at the hardware store). The quantities volt, ampere, and watt are all MKS units.

The corresponding CGS quantities are ***statvolts*** (1 statvolt = 300 volts), ***statamps*** (1 statamp = 3 billion amps) and ***ergs per second*** (1 erg = 10^{-7} joules), all quantities you have never heard of and do not want to use. For this reason we use MKS units in this text.

The choice for the universal electric force constant in the MKS system of units, a choice made official in 1946, is

$$\boxed{K\,(\text{MKS units}) \equiv 10^{-7}c^2 = 9.00 \times 10^9} \qquad (7)$$

where c is the speed of light $(3 \times 10^8 \text{m/s})$. While the choice $10^{-7}c^2$ appears rather arbitrary, it is the choice that gives us volts, amps and watts.

There is one more step in getting the standard form of Coulomb's law in MKS units. The universal electric constant K is written in the form

$$K \equiv \frac{1}{4\pi\varepsilon_0} \qquad (8)$$

so that Coulomb's law becomes

$$\boxed{F_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}} \quad \textit{Coulomb's electric force law} \qquad (9)$$

You would be quite justified in asking ***why did they do that?*** The answers are: the 4π was put into Coulomb's force law in order to get rid of a 4π that appears somewhere else (namely in our discussion of electric flux). And the constant ε_0 (epsilon naught) was put downstairs in order to make electrical theory in the 1800s look more like the theory of fluids.

We can use Equation (8) to solve for the numerical value of ε_0. We get

$$\varepsilon_0 \equiv \frac{1}{4\pi K} = \frac{1}{4\pi \times 10^{-7}c^2} = \frac{1}{4\pi \times 9 \times 10^9}$$

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12}} \quad \textit{standard electric force constant} \qquad (10)$$

THE COULOMB

With the choice of $K = 9.00 \times 10^9$ in the MKS system, the electric charge Q is measured in ***coulombs***. Conceptually, we like to think of how big a coulomb of charge is by counting the number of electrons it takes to make up a coulomb of them. We discuss this in the next section.

CHARGE ON THE ELECTRON

The twentieth century has given us a fundamental definition of a unit charge, namely the charge (e) on an electron. The quantity (e) is actually the charge on the proton since the electron, by Franklin's definition, is negative. But we still call the positive number (e) the ***charge on the electron***. In 1910, Robert Millikan, studying the motion of charged drops in a fine mist of oil, was able to measure the charge on an individual electron. The result is

$$\boxed{e = 1.60 \times 10^{-19}\text{coulombs}} \quad \textit{charge on an electron} \qquad (11)$$

From this we can immediately calculate how many electrons are in a coulomb of electrons. Using dimensions we have

$$e = 1.60 \times 10^{-19}\frac{\text{coulombs}}{\text{electron}}$$

$$\frac{1}{e} = 6.25 \times 10^{18}\,\frac{\text{electrons}}{\text{coulomb}} \qquad (12)$$

Thus in the MKS system the unit charge, one coulomb, turns out to be the amount of charge one would have in a bucket of 6.25×10^{18} electrons (again forgetting the minus sign).

Just as a liter of water is a large convenient collection of water molecules, 3.34×10^{25} of them, the coulomb can be thought of as a large collection of electron charges, 6.25×10^{18} of them.

THE HYDROGEN ATOM

The one place that Coulomb's law should apply, without the problem of dealing with a complex charge distribution, is in the case of the hydrogen atom. There we have a nucleus consisting of one proton of charge +e and one electron of charge –e. Since the proton is 1863 times as massive as the electron, the proton should sit at the center of the atom while the electron orbits around it. The force attracting the electron to the proton is $F_e = Ke^2/r^2$, and because of the analogy to a planet orbiting the sun, we might expect the electron to execute a Kepler orbit about the proton. This is suggested in Figure (3).

This does not happen, not because of a failure of the $1/r^2$ Coulomb's force law, but because of a failure of Newtonian mechanics. On an atomic scale the particle/wave nature of matter plays an increasingly important role, and must be taken into account in order to understand the behavior of the electron in hydrogen. Some classical ideas like conservation of energy and angular momentum still apply, but precise predictions of the electron behavior result only when we include the wave nature of the electron. We do this in detail in the quantum mechanics chapters.

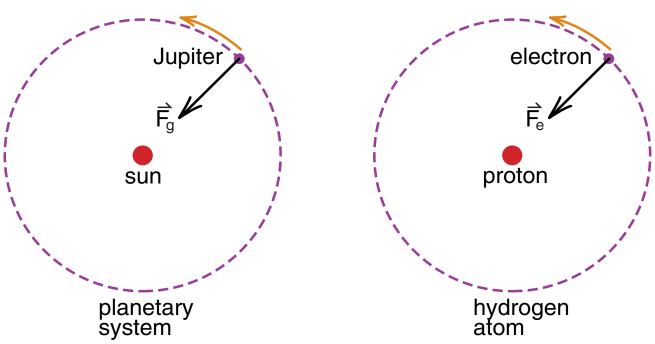


Figure 3
Classical picture of the hydrogen atom.

Molecular Forces

For now we will account for the wave nature of the electron by picturing the electron forming a negatively charged cloud surrounding the positive proton. For a neutral isolated hydrogen atom the cloud is spherical in shape, is most intense at the proton, and dies off in about 10^{-10} meters, which is called one angstrom, or Å, as indicated in Figure (4).

In quantum mechanics, the density of the electron cloud at some point is interpreted as the probability of finding the electron there. For now we will picture the electron cloud as simply a cloud of negative charge. In a complete hydrogen atom, the cloud contains just as much negative charge as the proton has positive charge, so that the atom is electrically neutral.

If you look carefully at the atoms in a bottle of room temperature hydrogen gas, you would not find neutral hydrogen atoms. Instead the atoms will be found in pairs called hydrogen ***molecules***, denoted by the chemists as H_2. The chemists would say that the hydrogen atoms are held together by a ***covalent bond***, which is an example of the ***molecular forces*** that hold atoms together to form molecules, crystals, and most of the matter around us. What we can do now, with Coulomb's law and the picture of an electron cloud, is to see how the hydrogen covalent bond is formed.

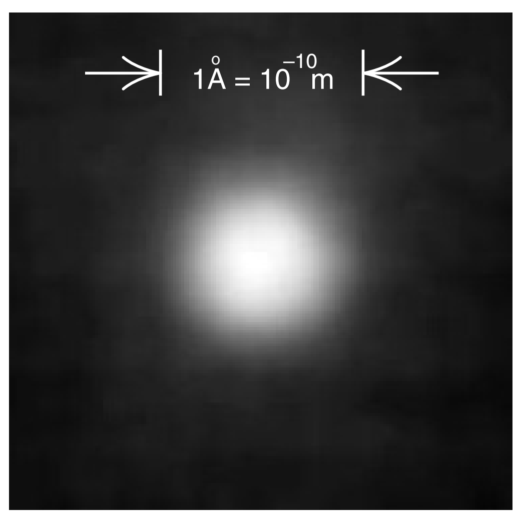


Figure 4
Cloud model of hydrogen atom.

HYDROGEN MOLECULE ION

To construct the hydrogen molecule, imagine that we start with a complete hydrogen atom and a single proton as indicated in Figure (5a). Here we are representing the electron cloud in the atom by an electron moving around to more or less fill a spherical region around its proton. In this case the external proton is attracted to the sphere of negative charge by a force that is just as strong as the repulsion from the hydrogen nucleus. As a result there is very little net force between the external proton and the neutral atom.

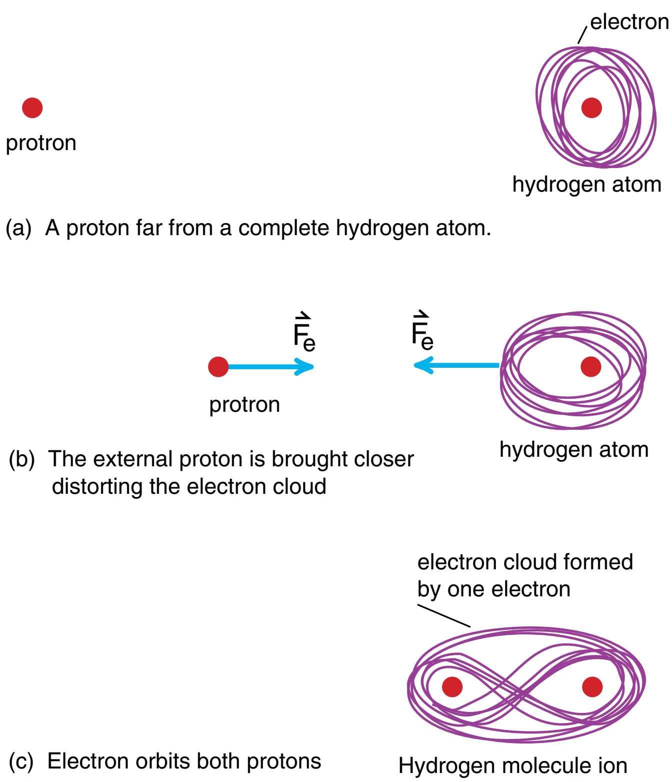


Figure 5a
Formation of a hydrogen molecule ion. To visualize how a hydrogen molecule ion can be formed, imagine that you bring a proton up to a neutral hydrogen atom.

Figure 5b
When the proton gets close, it distorts the hydrogen electron cloud. Since the distorted cloud is closer to the external proton, there is a net attractive force between the proton and the distorted hydrogen atom.

Figure 5c
If the protons get too close, they repel each other. As a result there must be some separation where there is neither attraction or repulsion. This equilibrium separation for the protons in a hydrogen molecule ion is 1.07 Angstroms. (1 Angstrom = 10^{-10} m.)

Now bring the external proton closer to the hydrogen atom. The electron cloud is now beginning to feel the attraction of external proton as well as its own proton. The result is that the cloud is distorted, sucked over a bit toward the external proton. The attractive force between the cloud and the external proton is now slightly greater than the repulsion between the protons. The external proton is now attracted to the neutral hydrogen atom for much the same reason that a charged rod attracted a neutral aluminium foil ball in our electrostatic experiment (shown in Figure (2a) reproduced here.

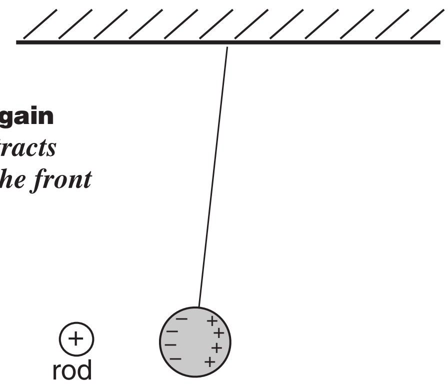


Figure 2a again
The + rod attracts electrons to the front side.

If we let go of the proton in Figure (5b), it will be sucked into the hydrogen atom. When it enters the electron cloud and gets close to the other proton, the repulsion between the protons can begin to exceed the attraction of the negative cloud. There is an intermediate position where the electron attraction and proton repulsion cancel and in Figure (5c) we have a stable object called a ***hydrogen molecule ion***. In the ion, the electron cannot tell the difference between the two protons, and thus forms a symmetric cloud about both. It is this shared electron cloud that forms the covalent bond between the two protons.

Hydrogen Molecule

Since the hydrogen molecule ion has two protons and only one electron, the total charge is +e and the ion can attract another electron into the cloud. Now we have a complete hydrogen molecule as indicated in Figure (6). This electron cloud with two electrons forms what is called a ***double ionic bond***, which is even stronger than the ***single bond*** of the hydrogen molecule ion.

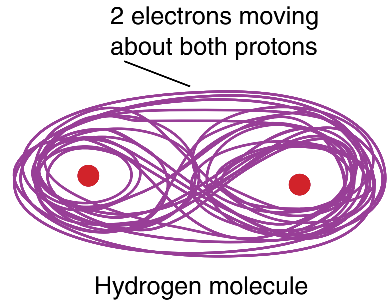


Figure 6
The hydrogen molecule ion of Figure (5c) has a net positive charge +e, and therefore can attract and hold one more electron. In that case both electrons orbit both protons and we have a complete hydrogen molecule. The equilibrium separation expands to 1.48 Angstroms.

CONSERVATION OF ELECTRIC CHARGE

In the early 1930s, our view of the basic structure of matter was fairly simple. Matter was made up of atoms, and atoms consisted of nuclei made up of protons and neutrons. Electrons surrounded the nucleus, and a complete neutral atom contained equal numbers of protons and electrons.

Then in 1933 Carl Anderson at Caltech discovered a particle of the same mass as the electron but of the opposite sign. Dirac had already formulated a theory for this new particle. He had a relativistic wave equation for the electron, an equation that had two solutions. One solution was the familiar electron. The other solution, after some thought, turned out to be Anderson's particle which became known as the ***positron***. It soon became clear that all relativistic wave equations for elementary particles would have two solutions, and the second solution became known as the ***anti matter*** solution.

A special feature of anti matter is that a particle and its anti particle can ***annihilate*** each other leaving behind only some form of energy. But in any such annihilation electric charge is conserved, meaning that the anti particle has the opposite charge of the particle. In the 1960s physicists were able to create the negative anti particle of the proton. (A matter-anti matter pair of particles can also be created from energy).

The elementary particle picture of matter gradually increased in complexity. In the early 1930s the ***muon*** was discovered. That is the particle we studied in the muon lifetime movie. After a positive anti particle muon stopped in the block of plastic, it sat around for a while and then decayed into a positron and a neutrino. The neutrino is electrically neutral and the positron has the same + charge as the original muon. Thus electric charge was conserved when the muon decayed.

By the 1950s, the number of known "elementary" particles had increased dramatically. In 1947 the family of π mesons was discovered, then K mesons and λ particles. The count of elementary particles had increased to the order of 100 or so by 1960. All but the familiar proton and electron are unstable and quickly decay after being created. Despite all the complexity of the creation and decay of all these particles, one result was simple. ***In every creation or decay, electric charge was conserved***. Just as much charge entered a reaction as came out of it.

In 1961, Murray Gell-Mann of Caltech devised a scheme that provided an orderly ranking of the myriad of elementary particles. His scheme, called the *eightfold way*, organized the elementary particles in much the same way that Mendeleev's periodic table organized the 100 or so different kinds of elements. I was privileged to be at Gell-Mann's first seminar introducing that scheme.

A few years later, Gell-Mann and George Zweig independently (both were from Caltech but Zweig was away on sabbatical) discovered the reason for the symmetry seen in Gell-Mann's plan. In their theory there were two families of particles. There were the light particles called ***leptons*** which consisted of the electron, muon and neutrino. All the rest of the particles, the proton, the neutron, the π mesons and the rest of the 100 or so particles were made up of ***quarks*** (a name Gell-Mann gave them).

The most common quarks are given the rather mundane names: the ***up quark*** and the ***down quark***. The proton consists of two up quarks and one down quark, while the neutron consists of two down quarks and one up quark.

Charge on a Quark

The truly hard to believe feature of the quarks is their fractional electric charge. The up quark charge is (+2/3)e and the down quark charge is (–1/3)e. Thus the total charge on the proton is 2(2e/3) + (–e/3) = e as indicated in Figure (7). For the neutron, with two down quarks and one up quark, we have a charge 2(–e/3) + (2e/3) = 0. In neither case do we end up with a fractional charge.

There is a strict rule that ***quarks always form particles that have integer values of electric charge***. You have particles like the proton and neutron that are made up of three quarks, and the so called mesons that are made from quark-(anti quark) pairs. (An anti quark has the opposite charge from the corresponding quark.)

Why can't you violate this strict rule by simply pulling a single quark out of a proton? The answer lies in the nature of the force that hold the quarks together to form particles like the neutron and proton. This force, known as the ***strong nuclear force*** is quite unlike electricity or gravity. These latter two forces get weaker, as $1/r^2$, as we move two particles apart. In contrast the quark force gets stronger as we try to pull the quarks apart. (It's more like a spring force.)

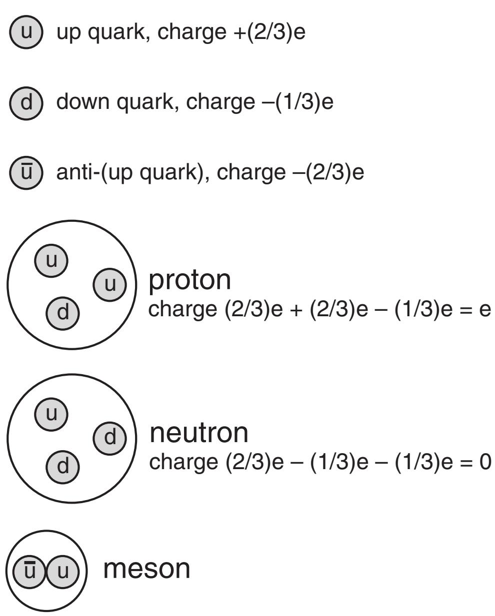


Figure 7
Some common quark particles.

Imagine, for example, that you tried to pull a quark out of a proton as shown in Figure (8a). The farther out you get it, the harder you have to pull. The increasing work you do goes into increasing the potential energy of the system. (Think of stretching a spring.)

At some point the energy you have added is enough to create a quark-(anti quark) pair as shown in Figure (8b). The quark in this pair goes back into the proton, while the anti quark joins the quark you pulled out to create a meson. Instead of a free quark, you end up with the proton and a meson as shown in Figures (8c,d). This process explains why mesons are created in profusion in particle accelerator experiments

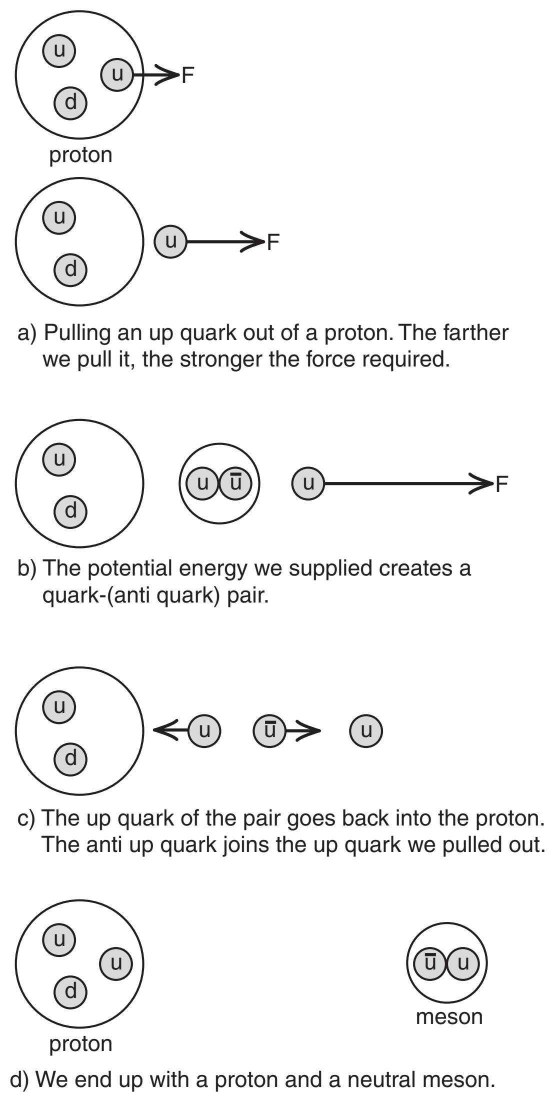


Figure 8
If we try to pull a single quark out of a proton or neutron, we get a meson instead. No one has ever seen a single quark.

The result of the peculiar increasing strength of the strong nuclear force is that quarks are trapped inside particles like protons, neutrons, and mesons, which are particles with integer values of electric charge.

The huge numbers of elementary particles seen in the 1950s, result from the fact that there are six kinds of quarks, the lightest being the common up and down quarks. The next lightest quark is given the weird name ***strange***. If you replace an up or down quark with a strange quark, you end up with what physicists used to call a ***strange particle***. (Gell-Mann is also responsible for the name "strange".)

EXERCISES WITH COULOMB'S LAW

The following four exercises are designed to give you some practice with Coulomb's law. In working these problems, use Coulomb's law in the form

$$F_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

where F_e is in newtons, Q_1 and Q_2 in coulombs, and r is in meters. The value of the force constants are

$$G = 6.67 \times 10^{-11} \frac{\text{newton meter}^2}{\text{kilogram}^2} \qquad (8\text{-}11)$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{coulomb}^2}{\text{newton meter}^2} \qquad (10\text{ again})$$

and the charge on the electron is

$$e = 1.60 \times 10^{-19}\text{coulombs} \qquad (11\text{ again})$$

In earlier work with projectiles, etc., it was often useful to keep track of your units during a calculation as a check for errors. In MKS electrical calculations, it is almost impossible to do so. Units like $coul^2/(meter^2 newton)$ are bad enough as they are. But if you look up ε_0 in a textbook, you will find its units are listed as *farads/meter*. In other words the combination $coul^2/(newton\ meter)$ was given the name *farad*. With naming like this, you do not stand a chance of keeping units straight during a calculation. You have to do the best you can to avoid mistakes without having the reassurance that your units check.

Example ***Two Charges***
Two positive charges, each 1 coulomb in size, are placed 1 meter apart. What is the electric force between them?

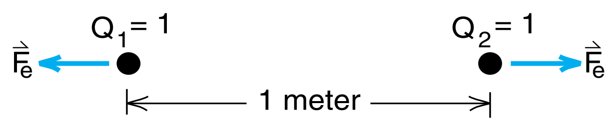


Figure 9

Solution: The force will be repulsive, and have a magnitude

$$\left|\vec{F}_e\right| = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}$$
$$= \frac{1}{4\pi\times 9 \times 10^{-12}} = 10^{10}\text{ newtons}$$

From the answer, 10^{10} newtons, we see that a coulomb is a huge amount of charge. We would not be able to assemble two 1 coulomb charges and put them in the same room. They would tear the room apart.

Exercise 2

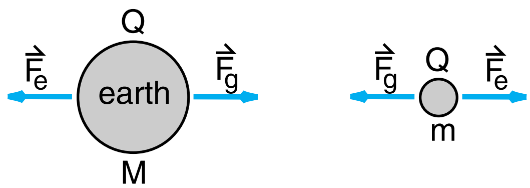


Figure 10

(a) Equal numbers of electrons are added to both the earth and the moon until the repulsive electric force exactly balances the attractive gravitational force. How many electrons are added to the earth and what is their total charge in coulombs?

(b) What is the mass, in kilograms of the electrons added to the earth in part (a)?

Exercise 3

Calculate the ratio of the electric to the gravitational force between two electrons. Why does your answer not depend upon how far apart the electrons are?

Exercise 4

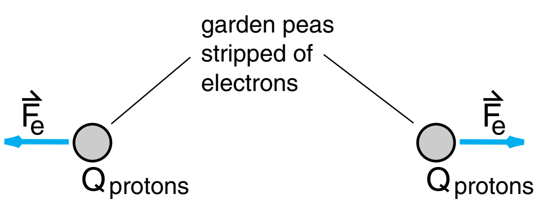


Figure 11

Imagine that we could strip all the electrons out of two garden peas, and then placed the peas one meter apart. What would be the repulsive force between them? Express your answer in newtons, and metric tons. (One metric ton is the weight of 1000 kilograms.) (Assume the peas each have about one Avogadro's number, or gram, of protons.)

Exercise 5

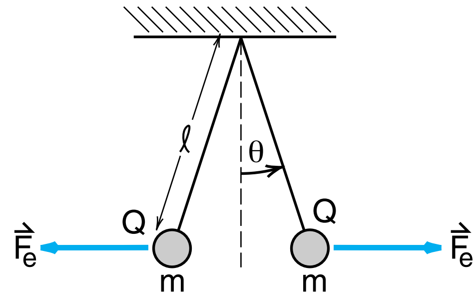


Figure 12

Two Styrofoam balls covered by aluminum foil are suspended by equal length threads from a common point as shown. They are both charged negatively by touching them with a rubber rod that has been rubbed by cat fur. They spread apart by an angle 2θ as shown. Assuming that an equal amount of charge Q has been placed on each ball, calculate Q if the thread length is $\ell = 40\,\text{cm}$, the mass m of the balls is m=10 gm, and the angle is $\theta = 5°$. Use Coulomb's law in the form $|\vec{F}_e| = Q_1 Q_2 / 4\pi\varepsilon_0 r^2$, and remember that you must use MKS units for this form of the force law.

APPENDIX
CLASSICAL MODEL OF HYDROGEN

(This material will be used when we get to the Bohr model of the hydrogen atom in Chapter 27.)

Despite the failure of Newton's second law at an atomic level, Neils Bohr found that a classical analysis of the hydrogen atom provided important clues as to the true nature of hydrogen. He considered a simple model in which the electron was moving in a circular orbit about the proton. Since the electron and proton both have a charge of magnitude e, the electric force $\vec{F}_e$ that the proton exerts on the electron is directed toward the proton and has a magnitude

$$F_e = \frac{Ke^2}{r^2} \qquad (13)$$

The electron, traveling in a circle of radius r, has an acceleration $\vec{a}$, also directed toward the proton, with a magnitude

$$a = \frac{v^2}{r} \qquad (14) \text{ also } (5\text{-}8)$$

Applying Newton's second law $\vec{F} = m\vec{a}$, and noting $\vec{F}$ and $\vec{a}$ are in the same directions, we get

$$F_e = ma; \quad \frac{Ke^2}{r^2} = \frac{mv^2}{r} \qquad (15)$$

One of the r's cancel and we immediately get the formula for the electron's kinetic energy

$$\frac{1}{2}mv^2 = \frac{Ke^2}{2r} \qquad (16)$$

The electron also has electrical potential energy. We use the convention that ***the potential energy is zero when the electron and proton are very far apart***. If we let the particles fall together they gain kinetic energy at the expense of electric potential energy. Since the potential energy started out at zero, it had to become ***negative*** as potential energy was lost. This is the same reason why the attractive gravitational force has negative potential energy.

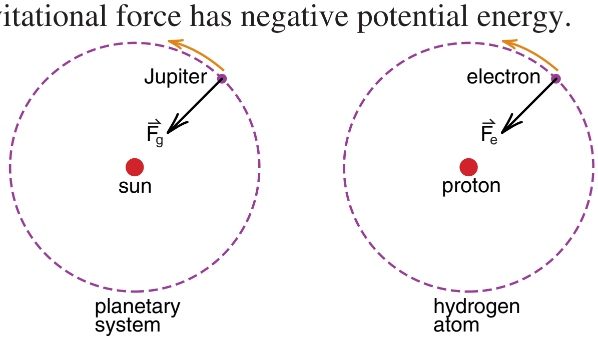


Figure 3 (repeated)
Classical picture of the hydrogen atom.

(We define gravitational potential energy as approaching zero when two masses are very far apart.)

The formula for the gravitational potential energy of particles of mass M_1 and M_2 separated by a distance r is

$$\left.\begin{array}{l}\text{Gravitational}\\ \text{potential}\\ \text{energy}\end{array}\right\} = -\frac{GM_1M_2}{r} \qquad (10\text{-}43)$$

which we discussed on page (10-19). It looks like the formula for the magnitude of the gravitational force, except that the r is not squared.

We did not derive this formula, but did show that it led to conservation of energy when we calculated satellite orbits.

Since we can go back and forth between Newton's law of gravity and Coulomb's electrical force law by replacing GM_1M_2 by KQ_1Q_2, we should expect that the formula for the electric potential energy of a proton and an electron should also be obtained from Equation (10-43) by replacing GM_1M_2 by $KQ_1Q_2 = Ke^2$ to get

$$\left.\begin{array}{l}\text{Electrical}\\ \text{potential}\\ \text{energy}\end{array}\right\} = -\frac{Ke^2}{r} \qquad (17)$$

Here is what may be a surprise. When an electron is in a circular orbit its negative electrical potential energy is twice as large as its positive kinetic energy of Equation (16). As a result the electron's total energy, kinetic plus potential energy is

$$\left.\begin{array}{l}\text{Total energy of}\\ \text{an electron in}\\ \text{a circular orbit}\end{array}\right\} = \begin{array}{l}\text{kinetic}\\ \text{energy}\end{array} + \begin{array}{l}\text{potential}\\ \text{energy}\end{array}$$

$$= \frac{Ke^2}{2r} - \frac{Ke^2}{r} \qquad (18)$$

$$= -\frac{Ke^2}{2r}$$

We see that the electron's total energy is negative, and becomes more negative as the electron's orbital radius becomes smaller.

Another way of thinking of the electron's negative total energy is that the electron is in an energy well of depth $Ke^2/2r$. To get the electron out of that well, i.e., to pull the electron away from the proton, requires that we supply an amount of energy $Ke^2/2r$. One can call $Ke^2/2r$ the ***binding energy*** of the electron.

CHAPTER 17 REVIEW

*In this chapter our focus was on the concept of electric charge and Coulomb's force law. We began with a few electrostatic experiments that Ben Franklin could have used to demonstrate that there are two kinds of electric charge, which he called **positive** and **negative** charge. Due to Franklin's choice, that the charge on a glass rod that had been rubbed by silk was a positive charge, it turns out that the protons inside an atomic nucleus positive and the electrons surrounding the nucleus are negative.*

Conservation of Charge

The key feature of electric charge is that it is conserved. This conservation law shows up most clearly in elementary particle reactions where one kind of particle can turn into other kinds of particles, but there is no change in the total electric charge in the reaction.

As an example, we are reminded that all particles have an anti particle, and that it is sometimes easy to create or annihilate particle-anti particle pairs. But the anti particle always has the opposite charge of the particle, thus no electric charge is gained or lost in these pair creations and annihilations.

*Until the 1970s, it was thought that the fundamental unit of electric charge is what we call the **charge on the electron** and designated by the letter "e". That is a slight misnomer because electrons are negative and have a charge –e. It is the proton that has a charge +e.*

*Somewhat of a shock was the discovery in the 1970s, that protons, neutrons, and other similar particles are made up of more fundamental particles called **quarks**. The two most common quarks are the **up quark** that has a charge +2/3 e, and the **down quark** that has a charge –1/3 e. A proton consists of two up quarks and one down quark for a total charge 2/3e + 2/3e – 1/3e = +e. The neutron is made up of one up quark and two down quarks for a total charge 2/3e – 1/3e – 1/3e = 0.*

*A special feature of the so called **strong nuclear force** that holds quarks together, is that you cannot pull a single quark out of a proton or neutron. Quarks either come in groups of 3, or in quark-anti quark pairs in which the total charge is either +e, 0, or –e.*

Coulomb's Law

Both electricity and gravity are $1/r^2$ *force laws, but the similarity essentially ends there. Both force laws can be written in the form*

$$F_{gravity} = \frac{GM_1M_2}{r^2}\ ; \quad F_{electricity} = \frac{KQ_1Q_2}{r^2}$$

where r is the separation either between masses M_1 *and* M_2*, or between charges* Q_1 *and* Q_2*.*

The unit of mass is defined by the one kilogram platinum block kept in Paris, France. The result is that the universal gravitational constant can be determined by measuring the gravitational force between two masses and using the equation $G = F_{gravity}r^2/M_1M_2$*.*

This approach does not work for electric forces because it is not possible to create copies of a standard electrical charge that can be passed around for calibrating electrical measurements. We saw why in our few electrostatic measurements. Charge is very mobile and can leak off of charged rods onto foil balls and into moist air.

Rather than trying to first establish a unit charge, and then solving for the electric constant K, what was done was to first define the electric constant K and then find some other way to figure out how big a unit charge is. In the MKS system of units, the choice of K is

$$K \equiv 10^{-7} \times c^2 = 9.00 \times 10^9 \qquad \text{MKS units}$$

The choice $K = 10^{-7}c^2$ *for MKS units looks rather arbitrary, but it leads to the practical quantities of amperes, volts, and watts.*

In MKS units there is another trick to deal with. A new constant ε_0 is defined by the equation

$$\varepsilon_0 \equiv \frac{1}{4\pi K} = 8.85 \times 10^{-12} \qquad (10)$$

The result that the electrical force law, known as Coulomb's law, becomes

$$F_{electric} = \frac{K\,Q_1\,Q_2}{r^2} = \frac{Q_1\,Q_2}{4\pi\varepsilon_0 r^2} \qquad (9)$$

Nature provides an excellent standard unit of charge, namely the electron charge e. It is a better standard than the unit mass in Paris, because everyone has plenty of exact copies. The problem is that it is hard to count electrons. If we could, we would find that the standard MKS unit of charge, called the ***coulomb****, is equivalent to the charge on 6.25×10^{18} electrons. With the sign corrected, we have*

$$1\ \text{coulomb} = 6.25 \times 10^{18} e \qquad \text{(from 12)}$$

Electricity and Gravity

There are two major differences between electric and gravitational forces. They are:

1) Electric forces tend to be far, far stronger than gravitational forces. This is so true for atomic particles like electrons and protons that gravity can be completely neglected on an atomic scale. Two of our included exercises are designed to show how much stronger electricity is than gravity.

2) Newtonian gravity is always attractive, while electric forces are both attractive and repulsive. The main consequences are that electric forces often cancel, while gravitational forces do not. In Exercise 4 we ask you to calculate the electric force between two garden peas one meter apart. If, for some reason we could remove all the electrons from both peas, the force, as you should find out, is ***10^{15} TONS****! Yet the actual electric forces between the peas so completely cancel that one might even be able to measure the gravitational force between the peas. [Exercise 4 will play a major role in our introduction to magnetic force, and should be worked now.]*

The fact that electric forces tend to cancel on scales larger than atoms and molecules, is why the much weaker, but non cancelling gravity, dominates the large scale structure of matter, like planetary systems and galaxies. This, despite the fact that both are $1/r^2$ forces.

Coulomb's Law

Standard introductory physics texts provide lots of exercises with Coulomb's law. We do not do that because such calculations are not particularly important. Most electricity problems are solved using the concept of an electric field rather than Coulomb's law directly. We will have plenty of opportunity to use electric fields in the following chapters.

What we want you to get from this chapter is an understanding of where the electrical constants like $4\pi\varepsilon_0$ came from, an understanding of the law of conservation of electric charge, and an appreciation of how electric and gravitational forces and force laws differ. We want these ideas to stand out without being covered with a blizzard of made up exercises.

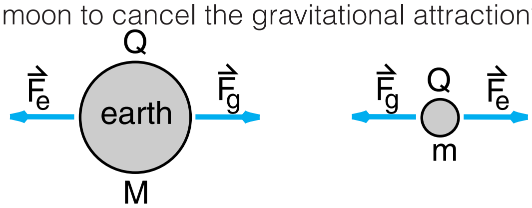
CHAPTER EXERCISES

Exercise 1 On page 3

Explain why the ball jumps away from the rod after the rod touches the ball.

Exercise 2 On page 13

How many electrons have to be added to the earth and moon to cancel the gravitational attraction?

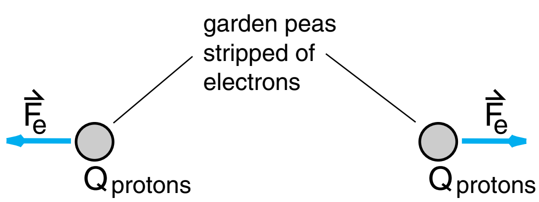


Exercise 3 On page 13

Calculate the ratio of the electric to the gravitational force between two electrons.

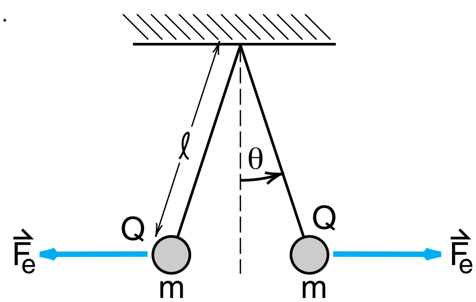
Exercise 4 On page 13

Calculate the electric force if we could strip all the electrons out of two garden peas.



Exercise 5 On page 13

Calculate the charge on two Styrofoam balls covered by aluminum foil, that are suspended by equal length threads.



REVIEW QUESTIONS

6) Explain how our electrostatic experiments with the foil covered balls and charged rods demonstrated that there are two kinds of electric charge.

7) The proton is made up of two up quarks of charge +2/3e and one down quark of charge –1/3e. What is an anti proton made up of? What are the charges of these particles and what is the total charge of the anti proton?

8) What happens to the electric charge when a proton and an anti proton annihilate?

9) Describe how two electrically neutral hydrogen atoms can end up attracting each other to form a hydrogen molecule.

10) Electric forces dominate the structure of atoms and molecules. The much weaker gravitational force dominates the structure of planets, stars, planetary systems, black holes, and galaxies. Why?