Chapter 31 non calculus

Quantum Mechanics I
Probability Interpretation

That light had both a particle and a wave nature became apparent with Einstein's explanation of the photoelectric effect in 1905. One might expect that such a discovery would lead to a flood of publications speculating on how light could behave both as a particle and a wave. But no such response occurred. The particle-wave nature was not looked at seriously for another 18 years, when de Broglie proposed that the particle-wave nature of the electron was responsible for the quantized energy levels in hydrogen. Even then there was great reluctance to accept de Broglie's proposal as a satisfactory thesis topic.

Why the reluctance? Why did it take so long to deal with the particle-wave nature, first of photons, then of electrons? What conceptual problems do we encounter when something behaves both as a particle and as a wave? How are these problems handled? That is the subject of the three quantum mechanics chapters which follow.

TWO SLIT EXPERIMENT

Of all the experiments in physics, it is perhaps the 2 slit experiment that most clearly, most starkly, brings out the problems encountered with the particle-wave nature of matter. For this reason we will use the 2 slit experiment as the basis for much of the discussion of this chapter.

Let us begin with a review of the 2 slit experiment for water and light waves. Figure (1) shows the wave pattern that results when water waves emerge from 2 slits. The lines of nodes are the lines along which the waves from one slit just cancel the waves coming from the other. Figure (2) shows our analysis of the 2 slit pattern. The path length difference to the first minimum must be half a wavelength $\lambda/2$. This gives us the two similar, shaded, triangles shown in Figure (2).

If y_{min} is much less than D, which it is for most 2 slit experiments, then the hypotenuse of the big triangle is approximately D, and equating corresponding sides of the similar triangles gives us the familiar relationship

$$\frac{\lambda/2}{d} = \frac{y_{min}}{D}$$

$$\boxed{\lambda = \frac{2y_{min}d}{D}} \qquad (1)$$

Figure (3a) is the pattern we get on a screen if we shine a laser beam through 2 slits. To prove that the dark bands are where the light from one slit cancels the light from the other, we have in Figure (3b) moved a razor blade in front of one of the slits. We see that the dark bands disappear, and we are left with a one slit pattern. The dark bands disappear because there is no longer any cancellation of the waves from the 2 slits.

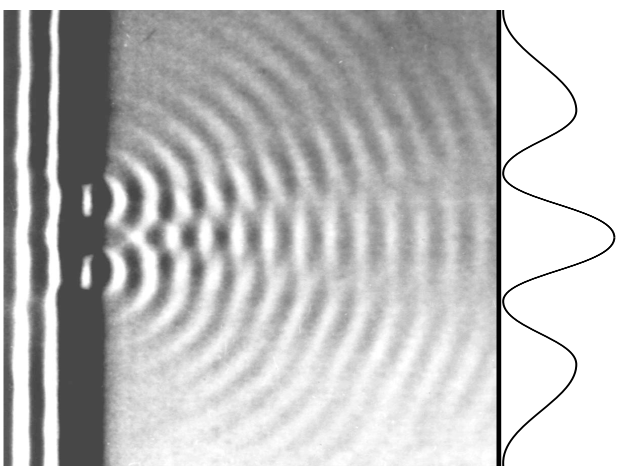


Figure 1
Water waves emerging from two slits.

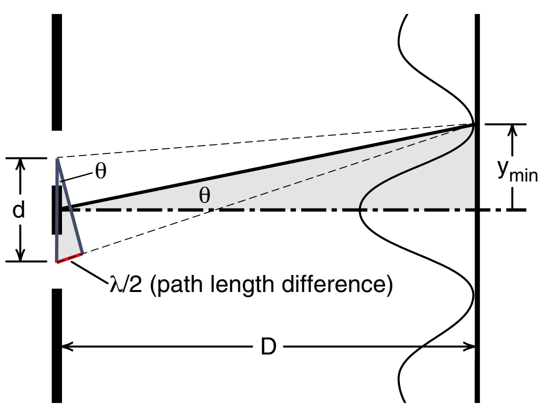


Figure 2
Analysis of the two slit pattern. We get a minimum when the path length difference is half a wavelength.

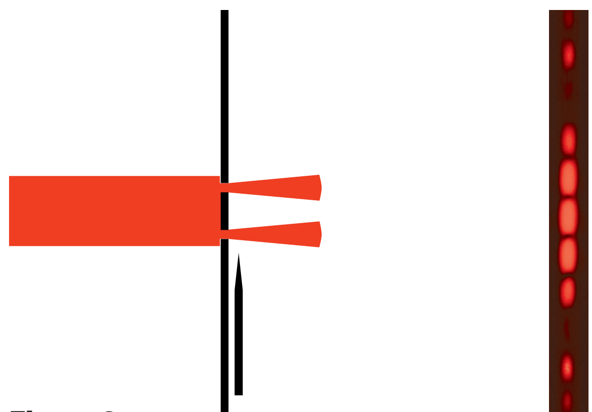


Figure 3a
Two slit interference pattern for light. The closely spaced dark bands are where the light from one slit cancels the light from the other.

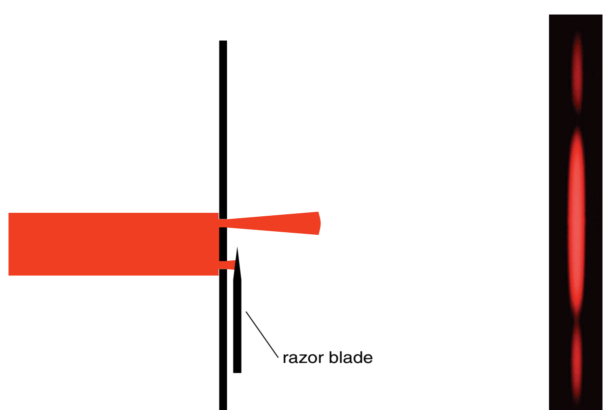


Figure 3b
Move a razor blade in front of one of the slits, and the closely spaced dark bands disappear. There is no more cancellation.

In 1961, Claus Jönsson did the 2 slit experiment using electrons instead of light, with the results shown in Figure (4). Assuming that the electron wavelength is given by the de Broglie formula $p = h/\lambda$, the dark bands are located where one would expect waves from the 2 slits to cancel. The 2 slit experiment gives the same result for light and electron waves.

The Two Slit Experiment from a Particle Point of View

In Figure (3a), the laser interference patterns were recorded on a photographic film. The pattern is recorded when individual photons of the laser light strike individual silver halide crystals in the film, producing a dark spot where the photon landed. Where the image is bright in the positive print, many photons have landed close together exposing many crystal grains.

In a more modern version of the experiment one could use an array of photo detectors to count the number of photons landing in each small element of the array. The number of counts per second in each detector could then be sent to a computer and the image reconstructed on the computer screen. The result would look essentially the same as the photograph in Figure (3a).

The point is that the image of the two slit wave pattern for light is obtained by counting particles, not by measuring some kind of wave height. When we look at the two slit experiment from the point of view of counting particles, the experiment takes on a new perspective.

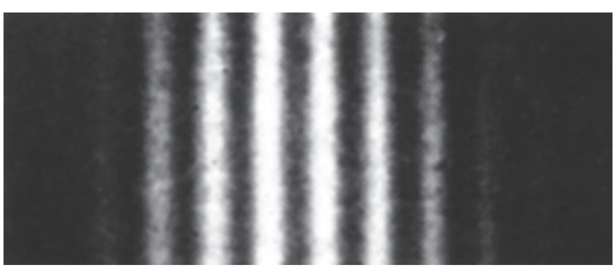


Figure 4
Two slit experiment using electrons. (By C. Jönsson)

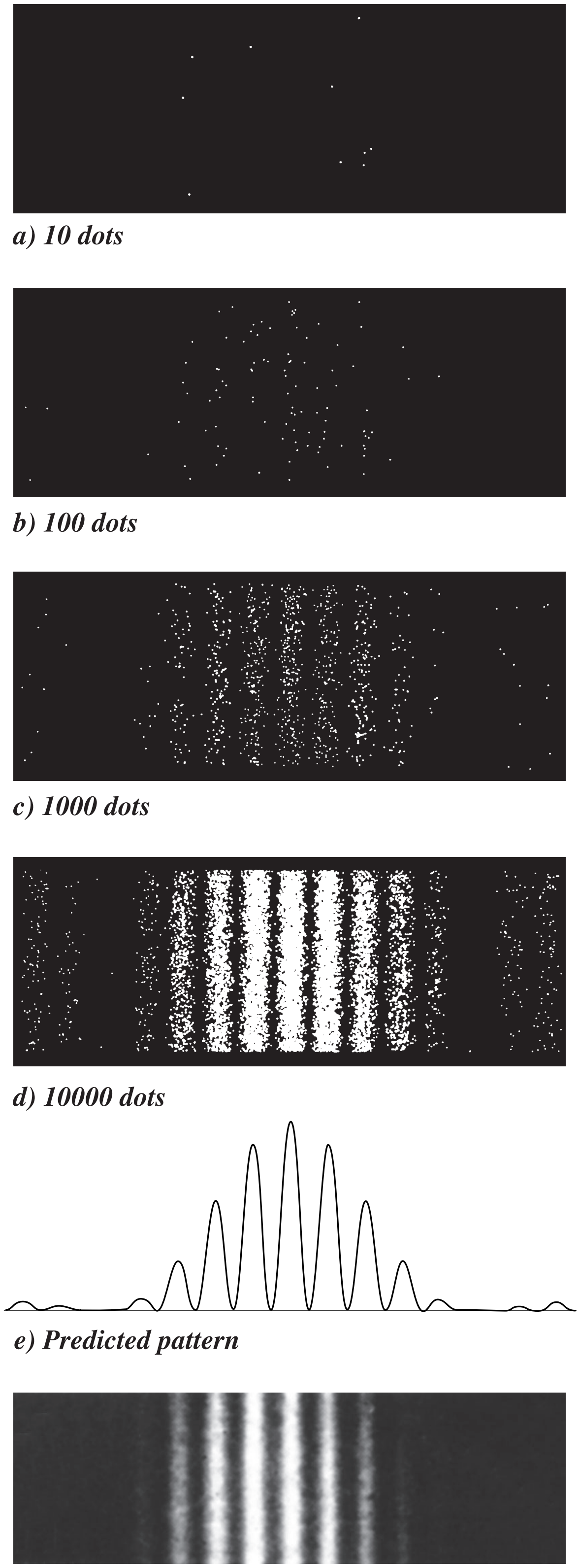
Imagine yourself shrunk down in size so that you could stand in front of a small section of the photographic screen in Figure (3a). Small enough that you want to avoid being hit by one of the photons on the laser beam. As you stand at the screen and look back at the slits, you see photons being sprayed out of both slits as if two machine guns were firing bullets at you, but you discover that there is a safe place to stand. There are these dark bands where the particles fired from one slit cancel the particles coming from the other.

When one of the slits is closed, there is no more cancellation, the dark bands disappear as seen in Figure (3b). There is no safe place to stand when particles are being fired at you from only one slit. It is hard to imagine in our large scale world how it would be safe to have two machine guns firing bullets at you, but be lethal if only one is firing. It is hard to visualize how machine gun bullets could cancel each other. But the particle-wave nature of light seems to require light particles to do so. No wonder the particle nature of light remained an enigma for nearly 20 years.

Two Slit Experiment— One Particle at a Time

You might object to our discussion of the problems involved in interpreting the two slit experiment. After all, Figure (1) shows water waves going through two slits and producing an interference pattern. The waves from one slit cancel the waves from the other at the lines of nodes. Yet water consists of particles—water molecules. If we can get a two slit pattern for water molecules, what is the big deal about getting a two slit pattern for photons ? Couldn't the photons somehow interact with each other the way water molecules do, and produce an interference pattern?

Photons do not interact with each other the way water molecules do. Two laser beams can cross each other with no detectable interaction, while two streams of water will splash off of each other. But one still might suspect that the cancellation in the two slit experiment for light is caused by some kind of interaction between the photons. This is even more likely in the case of electrons, which are strongly interacting charged particles.



a) 10 dots

b) 100 dots

c) 1000 dots

d) 10000 dots

e) Predicted pattern

f) Experimental results by C. Jönsson

Figure 5
Computer simulation of the 2 slit electron diffraction experiment, as if the electrons had landed one at a time.

In an earlier text, we discussed the possibility of an experiment in which electrons would be sent through a two slit array, one electron at a time. The idea was to eliminate any possibility that the electrons could produce the two slit pattern by bouncing into each other or interacting in any way. Since the experiment had not yet been done, we drew a sketch of what the results should look like. That sketch appears in several introductory physics texts.

When he saw the sketch, Lawrence Campbell of the Los Alamos Scientific Laboratories did a computer simulation of the experiment. We will first discuss Campbell's simulation, and then compare the simulation with the results of the actual experiment which was performed 20 years later in 1989.

It is not too hard to guess some of the results of sending electrons through two slits, one at a time. After the first electron goes through, you end up with one dot on the screen showing where the electron hit. The single dot is not a wave pattern. After two electrons, two dots; you cannot make much of a wave pattern out of two dots.

If, after many thousands of electrons have hit the screen, you end up with a two slit pattern like that shown in Figure (5f), that means that none of the electrons land where there will eventually be a dark band. You know where the first dot, and the second dot, cannot be located. Although two dots do not suggest a wave pattern, some aspects of the wave have already imposed themselves by preventing the dots from being located in a dark band.

To get a better idea of what is happening, let us look at Campbell's simulation in Figure (5). In (5a), and (5b) we see 10 dots and 100 dots respectively. In neither is there an apparent wave pattern, both look like a fairly random scatter of dots. But by the time there are 1000 dots seen in (5c), a fairly distinctive interference pattern is emerging. With 10,000 dots of (5d), we see a close resemblance between Campbell's simulation and Jönsson's experimental results. Figure (5e) shows the wave pattern used for the computer simulation.

Although the early images in Figure (5) show nearly random patterns, there must be some order. Not only do the electrons not land where there will be a dark band, but they must also accumulate in greater numbers where the brightest bands will eventually be. If this were a roulette type of game in Las Vegas, you should put your money on the center of the brightest band as being the location most likely to be hit by the next electron.

Campbell's simulation was done as follows. Each point on the screen was assigned a probability. The probability was set to zero at the dark bands and to the greatest value in the brightest band. Where each electron landed was randomly chosen, but a randomness governed by the assigned probability.

How to assign a probability to a random event is illustrated by a roulette wheel. On the wheel, there are 100 slots, of which 49 are red, 49 black and 2 green. Thus where the ball lands, although random, has a 49% chance of being on red, 49% on black, 2% on green, and 0% on blue, there being no blue slots.

In the two slit simulation, the probability of the electron landing at some point was proportional to the intensity of the two slit wave pattern at that point. Where the wave was most intense, the electron is most likely to land. Initially the pattern looks random because the electrons can land with roughly equal probability in any of the bright bands. But after many thousands of electrons have landed, you see the details of the two slit wave pattern. The dim bands are dimmer than the bright ones because there was a lower probability that the electron could land there.

Figure (6) shows the two slit experiment performed in 1989 by Akira Tonomura and colleagues. The experiment involved a novel use of a superconductor for the two slits, and the incident beam contained so few electrons per second that no more than one electron was between the slits and the screen at any one time. The screen consisted of an array of electron detectors which recorded the time of arrival of each electron in each detector. From this data the researchers could reconstruct the electron patterns after 10 electrons (6a), 180 electrons (6b), 3000 electrons (6c), 20,000 electrons (6d) and finally after 70,000 electrons in Figure (6e). Just as in Campbell's simulation, the initially random looking patterns emerge into the full two slit pattern when enough electrons have hit the detectors.

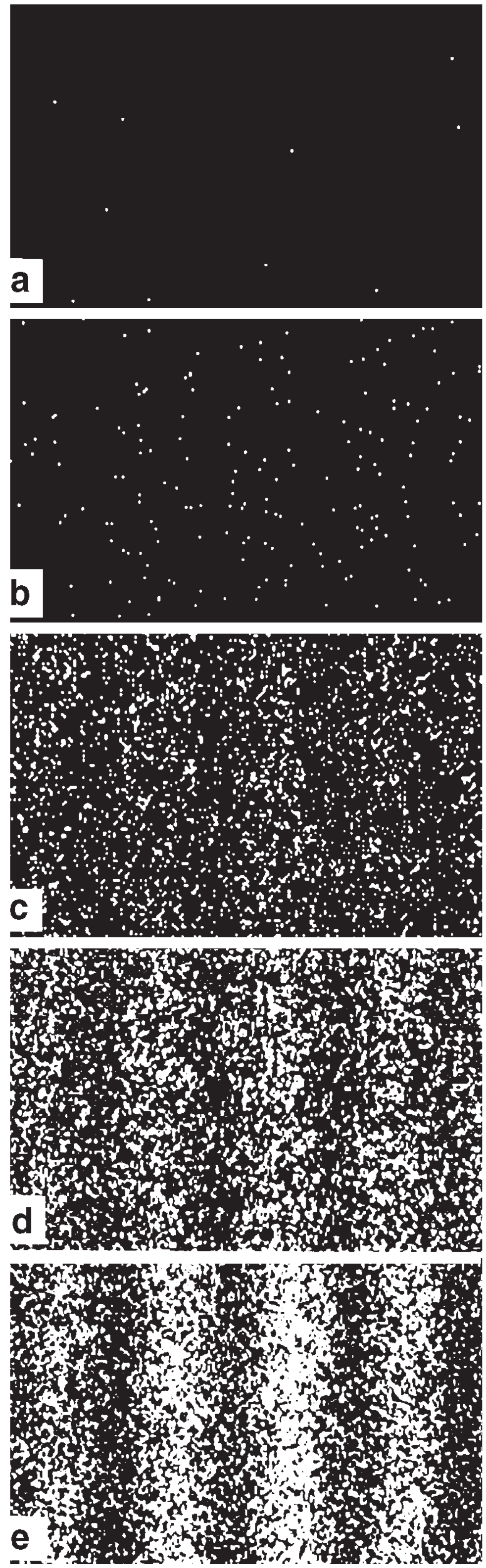


Figure 6
Experiment in which the 2 slit electron interference pattern is built up one electron at a time. (A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, American Journal of Physics, Feb. 1989. See also Physics Today, April 1990, Page 22.)

BORN'S INTERPRETATION OF THE PARTICLE WAVE

In 1926, while calculating the scattering of electron waves, Max Born discovered an interpretation of the electron wave that we still use today. In Born's picture, the electron is actually a particle, but it is the electron wave that governs the behavior of the particle. The electron wave is a probability wave governing the probability of where you will find the electron.

To apply Born's interpretation to the two slit electron experiment, we do what Campbell did in the simulation of Figure (5). We first calculate what the wave pattern at the screen would be for a wave passing through the two slits. It is the two slit interference pattern we have seen for water waves, light waves and electron waves. ***We then interpret the intensity of the pattern at some point on the screen as being proportional to the probability that the electron will land at that point.*** We cannot predict where any given electron will actually land, any more than we can predict where the ball will end up on the roulette wheel. But we can predict what the pattern will look like after many electrons have landed. If we repeat the experiment, the electrons will not land in the same places, but eventually the same two slit pattern will result.

Exercise 1

Figure (29-16) reproduced here, shows the diffraction pattern produced when a beam of electrons is scattered by the atoms of a graphite crystal. Explain what you would expect to see if the electrons went through the graphite crystal one at a time and you could watch the pattern build up on the screen. Could you market this apparatus in Las Vegas, and if so, how would you use it?

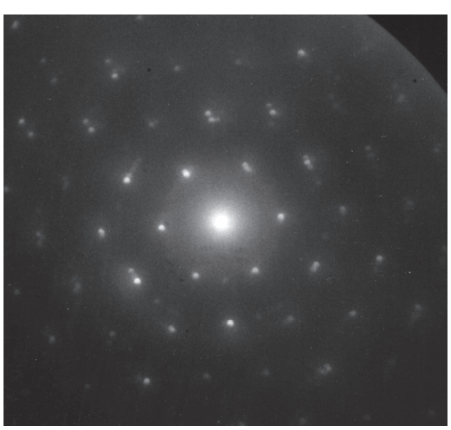


Figure 29-16
Diffraction pattern produced by electrons passing through a graphite crystal.

PHOTON WAVES

Both electrons and photons have a particle-wave nature related by the de Broglie formula $p = h/\lambda$, and both produce a two slit interference pattern. Thus one would expect that the same probability interpretation should apply to electron waves and light waves.

We have seen, however, that a light wave, according to Maxwell's equations, consists of a wave of electric and magnetic fields $\vec{E}$ and $\vec{B}$. These are vector fields that at each point in space have both a magnitude and a direction. Since probabilities do not point anywhere, we cannot directly equate $\vec{E}$ and $\vec{B}$ to a probability.

To see how to interpret the wave nature of a photon, let us first consider something like a radio wave or a laser beam that contains many billions of photons. In our discussion of capacitors in Chapter 22, we saw that the energy density in a classical electric field was given by

$$\left.\begin{matrix}\textit{energy density in}\\ \textit{an electric field}\end{matrix}\right\} = \frac{\varepsilon_0 E^2}{2} \qquad (22\text{-}37)$$

where $E^2 = \vec{E}\cdot\vec{E}$. In an electromagnetic wave there are equal amounts of energy in the electric and the magnetic fields. Thus the energy density in a classical electromagnetic field is twice as large as that given by Equation (22-37), and we have

$$\left.\begin{matrix}\textit{energy density in an}\\ \textit{electromagnetic wave}\end{matrix}\right\} = \varepsilon_0 E^2 \, \frac{\text{joules}}{\text{meter}^3}$$

If we now picture the electromagnetic wave as consisting of photons whose energy is given by Einstein's photoelectric formula

$$E_{photon} = hf \, \frac{\text{joules}}{\text{photon}}$$

then the density of photons in the wave is given by

$$n = \frac{\varepsilon_0 E^2 \,\text{joules/meter}^3}{hf \;\;\text{joules/photon}}$$

$$n = \frac{\varepsilon_0 E^2}{hf}\,\frac{\text{photons}}{\text{meter}^3} \quad \begin{matrix}\textit{density of photons}\\ \textit{in an electromagnetic}\\ \textit{wave of frequency f}\end{matrix} \qquad (1)$$

where f is the frequency of the wave.

(In Exercise 2, we have you estimate the density of photons one kilometer from the antenna of the student AM radio station at Dartmouth College. The answer is around .25 billion photons per cm^3— so many photons that it would be hard to detect them individually.)

Exercise 2

To estimate the density of photons in a radio wave, we can, instead of calculating $\vec{E}$ for the wave, simply use the fact that we know the power radiated by the station. As an example, suppose that we are one kilometer away from a 1000 watt radio station whose frequency is $1.4 \times 10^6 Hz$. A 1000 watt station radiates 1000 joules of energy per second or 10^{-6} joules in a nanosecond. In one nanosecond the radiated wave moves out one foot or about 1/3 of a meter. If we ignore spatial distortions of the wave, like reflections from the ground, etc., then we can picture this 10^{-6} joules of energy as being located in a spherical shell 1/3 of a meter thick, expanding out from the antenna.

(a) What is the total volume of a spherical shell 1/3 of a meter thick and 1 kilometer in radius?

(b) What is the average density of energy, in joules/m^3 of the radio wave 1 kilometer from the antenna?

(c) What is the energy, in joules, of one photon of frequency $1.4 \times 10^6 Hz$?

(d) What is the average density of photons in the radio wave 1 kilometer from the station? Give the answer first in photons/m^3 and then photons per cubic centimeter. (The answer should be about .25 billion photons/cm^3.)

Now imagine that instead of being one kilometer from the radio station, you were a million kilometers away. Since the volume of a spherical shell 1/3 of a meter thick increases as r^2, [the volume being $(1/3) \times 4\pi r^2$] the density of photons would decrease as $1/r^2$. Thus if you were 10^6 times as far away, the density of photons would be 10^{-12} times smaller. At one million kilometers, the average density of photons in the radio wave would be

$$\left.\begin{array}{l}\text{number of photons}\\ \text{per cubic centimeter}\\ \text{at 1 million kilometers}\end{array}\right\} = \frac{\text{number at 1km}}{10^{12}} = \frac{.25 \times 10^9}{10^{12}} = .00025\ \frac{\text{photons}}{\text{cm}^3}$$

In the classical picture of Maxwell's equations, the radio wave has a continuous electric and magnetic field even out at 1 million kilometers. You could calculate the value of $\vec{E}$ and $\vec{B}$ out at this distance, and the result would be sinusoidally oscillating fields whose structure is that shown back in Figure (24-5). But if you went out there and tried to observe something, all you would find is a few photons, on the order of .25 per liter (about one per gallon of space). If you look in 1 cubic centimeter of space, chances are you would not find a photon.

So how do you use Maxwell's equations to predict the results of an experiment to detect photons a million kilometers from the antenna? First you use Maxwell's equation to calculate $\vec{E}$ at the point of interest, then evaluate the quantity $(\varepsilon_0 E^2/hf)$, and finally interpret the result as the probability of finding a photon in the region of interest. If, for example, we were looking in a volume of one liter ($1000 cm^3$), the probability of finding a photon there would be about .25 or 25%.

This is an explicit prescription for turning Maxwell's theory of electromagnetic radiation into a probability wave for photons. If the wave is intense, as it was close to the antenna, then $(\varepsilon_0 E^2/hf)$ represents the density of photons. If the wave is very faint, then $(\varepsilon_0 E^2/hf)$ becomes the probability of finding a photon in a certain volume of space.

Exercise 3

The laser we used to create the patterns in Figure (3) is a 1 milliwatt (10^{-3} watts)laser. That means that the laser emits 10^{-3} joules of energy every second. The wavelength of the laser beam is 660 nanometers, as we found in Exercise (5) on page (25-13). How maqny photons are in 1 foot (1/3 meter) of the beam? (Hint: how long is the laser beam if you aim the laser up into a clear sky and turn it on for one second?)

REFLECTION AND FLUORESCENCE

An interesting example of the probability interpretation of light waves is provided by the phenomena of reflection and of fluorescence.

When a light beam is reflected from a metal surface, the angle of reflection, labeled θ_r in Figure (7a), is equal to the angle of incidence θ_i . The reason for this is seen in Figure (7b). The incident light wave is scattered by many atoms in the metal surface. The scattered waves add up to produce the reflected wave as shown in Figure (7b). Any individual photon in the incident wave must have an equal probability of being scattered by all of these atoms in order that the scattered probability waves add up to the reflected wave shown in (7b).

When you have a fluorescent material, you see a rather uniform eerie glow rather than a reflected wave. The light comes out in all directions as in Figure (8a).

The wavelength of the light from a fluorescent material is not the same wavelength as the incident light. What happens is that a photon in the incident beam strikes and excites an individual atom in the material. The excited atom then drops back down to the ground state radiating two or more photons to get rid of the excitation energy. (Ultraviolet light is often used in the incident beam, and we see the lower energy visible photons radiated from the fluorescing material.)

The reason that fluorescent light emerges in many directions rather than in a reflected beam is that an individual photon in the incident beam is absorbed by and excites one atom in the fluorescent material. There is no probability that it has struck any of the other atoms. The fluorescent light is then radiated as a circular wave from that atom, and the emerging photon has a more or less equal probability of coming out in all directions above the material.

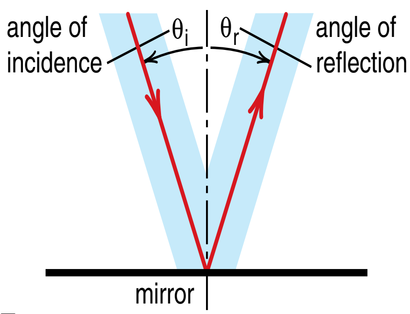


Figure 7a
When a light wave strikes a mirror, the angle of incidence equals the angle of reflection.

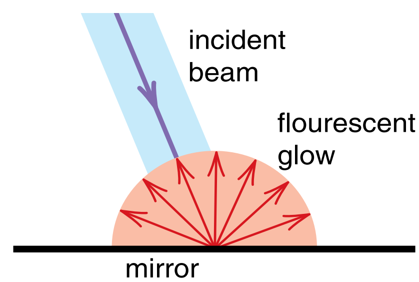


Figure 8a
When a beam of light strikes a fluorescent material, we see an eerie glow rather than a normal reflected light.

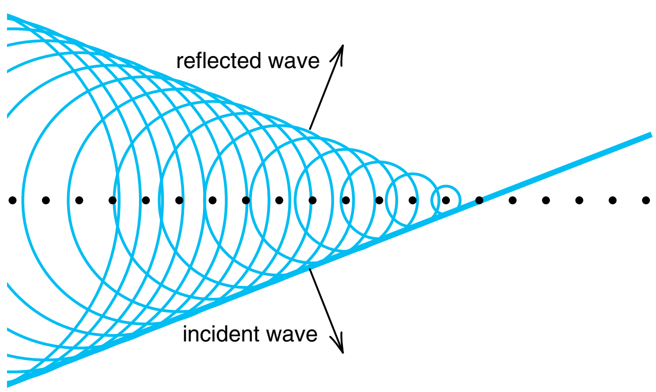


Figure 7b
The reflected wave results from the scattering of the incident wave by many atoms. If the incident wave contains a single photon, that photon must have an equal probability of being scattered by many atoms in order to emerge in the reflected wave.

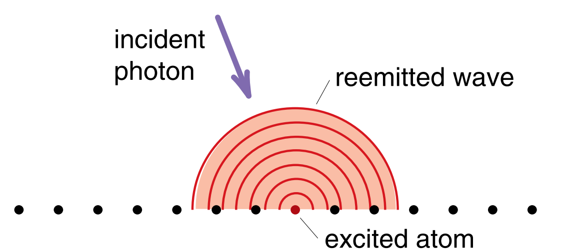


Figure 8b
Fluorescence occurs when an individual atom is excited and radiates its extra energy as two distinct photons. Since there is no chance that the radiation came from other atoms, the radiated wave emerges only from the excited atom.

CHAPTER 31 REVIEW

In a lecture demonstration we sent a laser beam through two closely spaced slits and observed the two slit interference pattern on a distant wall. In the pattern there is a series of regularly spaced dark bands which are caused by the cancellation of the waves from the two slits. We demonstrate this cancellation by sliding a razor blade in front of one of the slits as shown in Figure (3). The result is that the dark bands disappear and we see only the single slit diffraction pattern.

You can do a similar demonstration with water waves in a ripple tank. What were the dark bands in the laser beam demonstration are now the lines of nodes that emerge from the two slits as seen in Figure (25-9). Close one of the slits and the lines of nodes will disappear.

In 1961, Claus Jönsson sent a beam of electrons through two slits and got the interference pattern shown in Figure (4). The dark bands were located just where you would expect if the electrons behaved as a wave with a wavelength given by de Broglie's formula $p = h/\lambda$.

Did we feel that Jönsson's two slit electron wave pattern was a complete demonstration of the particle-wave nature of the electron? Not quite.

After all, water waves consist of particles, water molecules. And the two slit pattern in a ripple tank is not a demonstration of the wave nature of water molecules. Instead the two slit pattern results from the beams of waves from the two slits interacting with each other, cancelling at the lines of nodes. One could wonder if the dark lines in Jönsson pattern resulted from a similar interaction between beams of electrons emerging from the two slits.

To prevent such an interpretation, we proposed an experiment in which the electrons would be sent through the slits one at a time. In such an experiment there would be no possibility of the beams of electrons from one slit interacting with beams from the other slit. If, after many electrons had finally gone through, we still ended up with a two slit interference pattern, we would have direct experimental evidence that the electron's wave went through both slits even though the electron itself may have gone through only one of the slits. This would be a demonstration of the fundamental problem one faces with the particle-wave nature of matter.

When Jönsson did the electron two slit experiment, the equipment needed to do the experiment one electron at a time was not available. Thus for a textbook we drew sketches of what to expect as the electron interference pattern built one dot at a time. Initially we should see a random pattern of dots. After enough electrons have landed, the two slit pattern should emerge. The initial pattern cannot be completely random, because no electron can land where there will eventually be a dark band in the two slit pattern.

Larry Campbell at Los Alamos Laboratories cleaned up our crude sketches with the computer plots that are shown in Figure (5). He had the computer plot white dots one at a time, with a probability based program. The probability of a dot being drawn at any point on the screen was proportional to the intensity of the two slit pattern. There is a high probability of the electron or dot landing in one of the bright bands, and zero probability of it landing in the center of a dark band.

Twenty years later the experiment was actually performed with the results shown in Figure (6). There is an excellent match between the experimental results of Figure (6) and the probability analysis of Figure (5). This provides strong evidence supporting Max Born's interpretation the electron wave as a probability wave.

Review Continued
Quantum Mechanics

It is the probability interpretation of the particle wave that lies at the core of quantum mechanics. The rule for doing calculations in quantum mechanics is to first calculate the wave pattern you expect for some given experimental situation. That could be the two slit pattern we have just discussed, the wave pattern we described in our model atom of the last chapter or the electron wave patterns in atoms. Once you have solved for the wave pattern, you interpret the intensity of the wave pattern as being proportional to the probability of finding the particle at that point in the wave.

For example, the electron clouds which chemists call orbitals, are the calculated allowed standing wave patterns in the various atoms and molecules. Once the wave pattern has been calculated using Schroedinger's equation for electrons, you square the wave amplitude to get the intensity and interpret that to be the probability of finding an electron there. Basically this is how you do Quantum Mechanics.

CHAPTER EXERCISES

Exercise 1 0n page 6

Probability interpretation of the electron diffraction pattern produced by electrons passing through a graphite crustal.

Exercise 2 0n page 7

Calculating the density of photons in a radio wave.

Exercise 3 0n page 8

Calculating the number of photons in a one foot length of a laser beam.