Chapter 6 non calculus

Mass

By now we have learned how to use either calculus or the computer to predict the motion of an object whose acceleration is known. But in most problems we do not know the acceleration, at least initially. Instead we may know the forces acting on the object, or something about the object's energy, and use this information to predict motion. This approach, which is the heart of the subject of mechanics, involves mass, a concept which we introduce in this chapter.

In the metric system, mass is measured in grams or kilograms, quantities that should be quite familiar to the reader. It may be surprising that we devote an entire chapter to something that is measured daily by grocery store clerks in every country in the world. But the concept of mass plays a key role in the subject of mechanics. Here we focus on developing an experimental definition of mass, a definition that we can use without modification throughout our discussion of physics.

*After introducing the experimental definition, we will go through several experiments to determine how mass, as we defined it, behaves. In low speed experiments, the kind we can do using air tracks in demonstration lectures, the results are straightforward and are what one expects. But when we consider what would happen if similar experiments were carried out with one of the objects moving at speeds near the speed of light, we predict a very different behavior for mass. This new behavior is summarized by the Einstein mass formula, a strikingly simple result that one might guess, but which we cannot quite derive from the definition of mass, and the principle of relativity alone. What is needed in addition is the law of **conservation of linear momentum** which we will discuss in the next chapter.*

One of the striking features of Einstein's special theory of relativity is the fact that nothing, not even information, can travel faster than the speed of light. We can think of nature as having a speed limit c. In our world, speed limits are hard to enforce. We will see that the Einstein mass formula provides nature with an automatic way of enforcing its speed limit.

Einstein's mass formula appears to predict that no particle can quite reach the speed of light. We end the chapter with a discussion of how to handle particles, like photons and possibly neutrinos, that do travel at precisely the speed of light.

DEFINITION OF MASS

In everyday conversation the words ***mass*** and ***weight*** are used interchangeably. Physicists use the words mass and weight for two different concepts. Briefly, we can say that the weight of an object is the force that the object exerts against the ground, and we can measure weight with a device such as a bathroom scale. The weight of an object can change in different circumstances. For example, an astronaut who weighs 180 pounds while standing on the ground, floats freely in an orbiting space capsule. If he stood on a bathroom scale in an orbiting space craft, the reading would be zero, and we would say he is weightless. On the other hand the mass of the astronaut is the same whether he is in orbit or standing on the ground. An astronaut in orbit does not become massless. Mass is not what you measure when you stand on the bathroom scales.

What then is mass? One definition, found in the dictionary, describes mass as the property of a body that is a measure of the amount of material it contains. Another definition, which is closer to the one we will use, says that the more massive an object, the harder it is to budge.

Both of these definitions are too vague to tell us how to actually measure mass. In this section we will describe an experimental definition of mass, one that provides an explicit prescription for measuring mass. Then, using this prescription, we will perform several experiments to see how mass behaves.

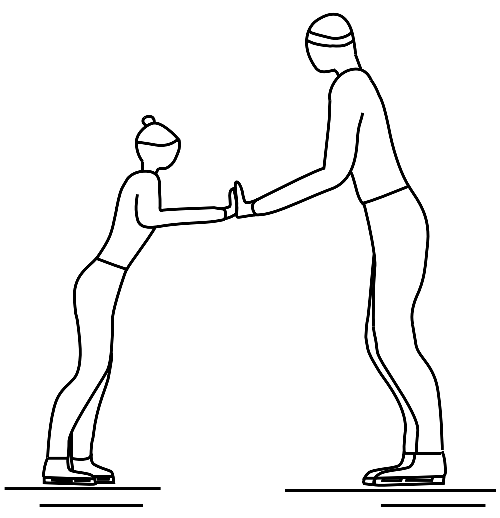


Figure 1
Two skaters, a father and a son, standing at rest on frictionless ice, push away from each other. The smaller, less massive child recoils faster than the more massive father.

Recoil Experiments

As a crude experiment suppose that the two skaters shown in Figure (1), a father and a child, stand in front of each other at rest and then push each other apart. The father hardly moves, while the child goes flying off. The father is more massive, harder to budge. No matter how hard or gently the skaters push apart, the big one always recoils more slowly than the smaller one. We will use this observation to define mass.

In a similar but more controlled experiment, we replace the skaters by two carts on what is called an ***air track.*** An air track consists of a long square metal tube with a series of small holes drilled on two sides as shown in Figure (2). A vacuum cleaner run backwards blows air into the tube, and the air escapes out through the small holes. The air carts have V-shaped bottoms which ride on a thin film of air, allowing the carts to move almost without friction along the track.

To represent the two skaters pushing apart on nearly frictionless ice, we set up two carts with a spring between them as shown in Figure (3a). A thread is tied between the carts to keep the spring compressed. When we burn the thread, the carts fly apart as shown in

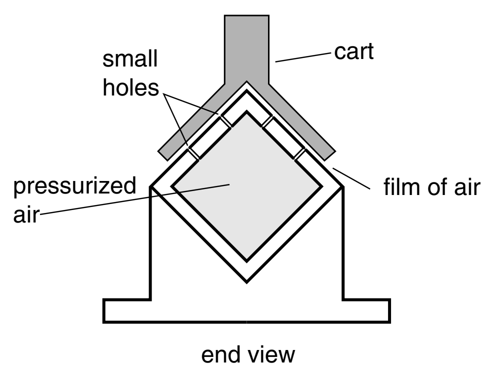


Figure 2
End view of an air track. Pressurized air from the back side of a vacuum cleaner is fed into a square hollow metal tube, and flows out through a series of small holes. A cart, riding on a film of air, can move essentially without friction along the track.

Figure (3b). If the two carts are made of similar material, but one is bigger than the other, the big one will recoil at lesser speed than the small one. We say that the big cart, the one that comes out more slowly, has more mass than the small one.

Because we can precisely measure the speeds v_A and v_B of the recoiling air carts, we can use the experiment pictured in Figures (3a,b) to define the mass of the carts. Let us call m_A and m_B the masses of carts A and B respectively. The simplest formula relating the masses of the carts to the recoil speeds, a formula that has the more massive cart recoiling at less speed is

$$\boxed{\frac{m_A}{m_B} = \frac{v_B}{v_A}} \qquad \textit{recoil definition of mass} \qquad (1)$$

In words, Equation 1 says that the ratio of the masses is inversely proportional to the recoil speeds. I.e., if m_A is the small mass, then v_B is the small speed.

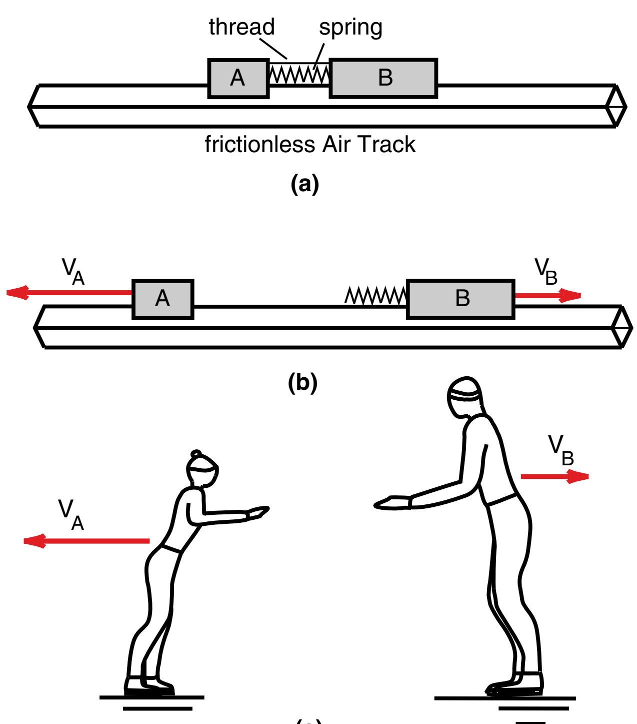


Figure 3
Recoil experiment. To simulate the two skaters pushing apart, we place two carts on an air track with a compressed spring between them. The carts are held together by a string. When the string is burned, the carts fly apart as did the skaters. The more massive cart recoils at a smaller speed $(v_B < v_A)$.

Properties of Mass

Since we now have an explicit prescription for measuring mass, we should carry out some experiments to see if this definition makes sense. Our first test is to see if the mass ratio m_A/m_B changes if we use different strength springs in the recoil experiment. If the ratio of recoil speeds v_B/v_A, and therefore the mass ratio, depends upon what kind of spring we use, then our definition of mass may not be particularly useful.

In the appendix to this chapter, we describe apparatus that allows us to measure the recoil speeds of the carts with fair precision. To within an experimental accuracy of 5% to 10% we find that the ratio v_B/v_A of the recoil speeds does not depend upon how hard the spring pushes the carts apart. When we use a stronger spring, both carts come out faster, in such a way that the speed ratio is unchanged. Thus to the accuracy of this experiment we conclude that the mass ratio does not depend upon the strength of the spring used.

Standard Mass

So far we have talked about the ratio of the masses of the two carts. What can we say about the individual masses m_A or m_B alone? There is a simple way to discuss the masses individually. What we do is select one of the masses, for instance m_B, as the ***standard mass***, and measure all other masses in terms of m_B. To express m_A in terms of the standard mass m_B, we multiply both sides of Equation (1) through by m_B to get

$$m_A = m_B \frac{v_B}{v_A} \qquad \textit{formula for } m_A \textit{ in terms of the standard mass } m_B \qquad (2)$$

For a standard mass, the world accepts that the platinum cylinder kept by the International Bureau of Weights and Measures near Paris, France, is precisely one kilogram. If we reshaped this cylinder into an air cart and used it for our standard mass, then we would have the following explicit formula for the mass of cart A recoiled from the standard mass.

$$m_A = \left(1 \text{ kilogram}\right) \times \left(\frac{v_{std}}{v_A}\right) \quad \begin{matrix}\textit{using the}\\ \textit{one kilogram}\\ \textit{cylinder for our}\\ \textit{standard mass}\end{matrix} \qquad (3)$$

where v_{std} is the recoil speed of the standard mass. Once we have determined the mass of one of our own carts, using the standard mass and Equation (3), we can then use that cart as our standard and return the platinum cylinder to the French.

Of course the French will not let just anybody use their standard kilogram mass. What they did was to make accurate copies of the standard mass, and these copies are kept in individual countries, one of them by the National Institute of Standards and Technology in Washington, DC which then makes copies for others in the United States to use.

Addition of Mass

Consider another experiment that can be performed using air carts. Suppose we have our standard cart of mass m_B, and two other carts which we will call C and D. Let us first recoil carts C and D from our standard mass m_B, and determine that C and D have masses m_C and m_D given by

$$m_C = m_B \frac{v_B}{v_C} \quad ; \quad m_D = m_B \frac{v_B}{v_D}$$

Now what happens if, as shown in Figure (4), we tie carts C and D together and recoil them from cart B. How is the mass m_{C+D} of the combination of the two carts related to the individual masses m_C and m_D? If we perform the experiment shown in Figure (4), we find that

$$m_{C+D} = m_C + m_D \quad \textit{mass adds} \qquad (4)$$

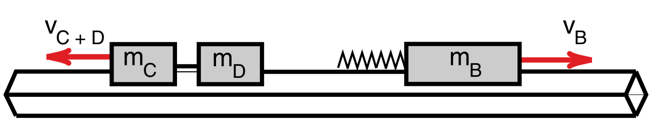


Figure 4
Addition of mass. If we tie two carts C and D together and recoil the pair from our standard mass m_A, and use the formula

$$m_{C+D} = m_B\left(\frac{v_B}{v_{C+D}}\right)$$

for the combined mass m_{C+D}, we find from experiment that $m_{C+D} = m_C + m_D$. In other words the mass of the pair of carts is the sum of the masses of the individual carts, or we can say that mass adds.

The experimental result, shown in Equation (4), is that mass adds. The mass of the two carts recoiled together is the sum of the masses of the individual carts. This is the reason we can associate the concept of mass with the quantity of matter. If, for example, we have two identical carts, then together the two carts have twice as much matter and twice as much mass.

Exercise 1

In physics labs, one often finds a set of brass cylinders of various sizes, each cylinder with a number stamped on it, representing its mass in grams. The set usually includes a 50-gm, 100-gm, 200-gm, 500-gm, and 1000-gm cylinder. Suppose that you were given a rod of brass and a hacksaw; describe in detail how you would construct a set of these standard masses. At your disposal you have a frictionless air track, two carts of unknown mass that ride on the track, the standard 1000-gm mass from France (which can be placed on one of the carts), and various things like springs, thread, and matches.

A Simpler Way to Measure Mass

The preceding problem illustrates two things. One is that with an air track, carts, and a standard mass, we can use our recoil definition to measure the mass of an object. The second is that the procedure is clumsy and rather involved. What we need is a simpler way to measure mass.

The simpler way involves the use of a balance, which is a device with a rod on a pivot and two pans suspended from the rod, as shown in Figure (5). If the balance is properly adjusted, we find from experiment that if equal masses are placed in each pan, the rod remains balanced and level. This means that if we place an unknown mass in one pan, and add brass cylinders of known mass to the other pan until the rod becomes balanced, the object and the group of cylinders have the same mass. To determine the mass of the object, all we have to do is add up the masses of the individual cylinders.

Inertial and Gravitational Mass

The pan balance of Figure (5) is actually comparing the downward gravitational force on the contents of the two pans. If the gravitational forces are equal, then the rod remains balanced. What we are noting is that there are equal gravitational forces on equal masses. This is an experimental result, not an obvious conclusion. For example, we could construct two air carts, one from wood and one from platinum. Keep adjusting the size of the carts until their recoil speeds are equal, i.e., until they have equal recoil masses. Then put these carts on the pan balance of Figure (5). Although the wood cart has a much bigger volume than the platinum one, we will find that the two carts still balance. The gravitational force on the two carts will be the same despite their large difference in size.

In 1922, the Swedish physicist Eötvos did some very careful experiments, checking whether two objects, which had the same mass from a recoil type of experiment would experience the same gravitational force as measured by a pan balance type of experiment. He demonstrated that we would get the same result to one part in a billion. In 1960, R. H. Dicke improved Eötvos' experiments to an accuracy of 1 part in 10^{11}.

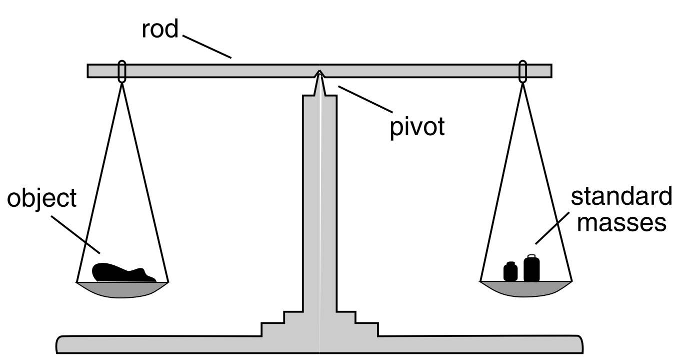


Figure 5
Schematic drawing of a pan balance. If the balance is correctly adjusted and if equal masses are placed in the pans, the rod will remain level. This allows us to determine an unknown mass simply by comparing it to a known one.

It is common terminology to call what we measure in a recoil experiment the ***inertial mass*** of the object, and what we measure using a pan balance the ***gravitational mass***. The experiments of Eötvos and Dicke demonstrate that inertial mass and gravitational mass are equivalent to each other to one part in 10^{11}. Is this a coincidence, or is there some fundamental reason why these two definitions of mass turn out to be equivalent? Einstein addressed this question in his formulation of a relativistic theory of gravity known as Einstein's ***General Theory of Relativity***. We will have more to say about that later.

Mass of a Moving Object

One reason we chose the recoil experiment of Figure (3) as our experimental definition of mass is that it allows us to study the mass of moving objects, something that is not possible with a pan balance.

From the air track experiments we have discussed so far, we have found two results. One is that the ratio of the recoil speeds, and therefore the ratio of the masses of the two objects, does not depend upon the strength of the spring or the individual speeds v_A and v_B. If we use a stronger spring so that m_A emerges twice as fast, m_B also emerges twice as fast so that the ratio m_A/m_B is unchanged.

In addition, we found that mass adds. If carts C and D have masses m_C and m_D when recoiled individually from cart B, then they have a combined mass $m_{C,D} = m_C + m_D$ when they are tied together and both recoiled from cart B.

RELATIVISTIC MASS

In our air track experiments, we found that the ratio of the recoil speeds did not depend upon the strength of the spring we used. However, when the recoil speeds approach the speed of light, this simple result can no longer apply. Because of nature's speed limit c, the ratio of the recoil speeds must in general change with speed.

To see why the recoil speed ratio must change, imagine an experiment involving the recoil of two objects of very different size, for example a bullet being fired from a gun as shown in Figure (6). Suppose, in an initial experiment not much gunpowder is used and the bullet comes out at a speed of 100 meters per second and the gun recoils at a speed of 10 cm/sec = .1 m/sec. For this case the speed ratio is 1000 to 1 and we say that the gun is 1000 times as massive as the bullet.

In a second experiment we use more gun powder and the bullet emerges 10 times faster, at a speed of 1000 meters per second. If the ratio of 1000 to 1 is maintained, then we predict that the gun should recoil at a speed of 1 meter per second. If we did the experiment, the prediction would be true.

But, as a thought experiment, imagine we used such powerful gun powder that the gun recoiled at 1% the speed of light. If the speed ratio remained at 1000 to 1, we would predict that the bullet would emerge at a speed 10 times the speed of light, an impossible result. The bullet cannot travel faster than the speed of light, the speed ratio cannot be greater than 100 to 1, and thus the ratio of the masses of the two objects must have changed.

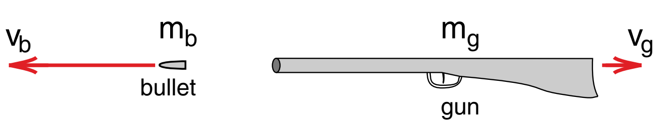


Figure 6
To discuss higher speed recoils, consider a bullet being fired from a gun. We are all aware that the bullet emerges at a high speed, but the gun itself also recoils. (The recoil of the gun becomes obvious the first time you fire a shotgun.) In this setup, the gunpowder is analogous to the spring, and the gun and bullet are analogous to the two carts.

In the next section we will discuss experiments in which, instead of a bullet being fired by a gun, an electron is ejected by an atomic nucleus. The electron is such a small particle that it is often ejected at speeds approaching the speed of light. The nuclei we will consider are so much more massive that they recoil at low speeds familiar to us, speeds like that of a jet plane or earth satellite. At these low speeds the mass of an object does not change noticeably with speed. Thus in these electron recoil experiments, the mass of the nuclei is not changing due to its motion. Any change in the ratio of recoil speeds is due to a change in the mass of the electron as the speed of the electron approaches the speed of light.

We will see that as we push harder and harder on the electron, trying to make it go faster than the speed of light, ***the mass of the electron increases instead***. It is precisely this increase in mass that prevents the electron emerging at a speed greater than the speed of light and this is how nature enforces the speed limit c.

Beta (β) Decay

The electron recoils we just mentioned occur in a process called β (beta) decay. In a β decay, a radioactive or unstable nucleus transforms into the nucleus of another element by ejecting an electron at high speeds as illustrated in Figure (7). In the process the nucleus itself recoils as shown.

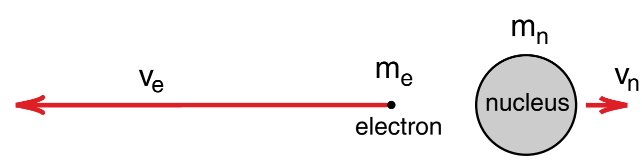


Figure 7
Radioactive decay of a nucleus by β decay. In this process the unstable nucleus ejects an electron, often at speeds v_e near the speed of light.

It turns out that in a β decay process, not only is the electron emitted, a particle called the ***neutrino*** is also emitted. (In the muon lifetime experiment, when the muon decayed, causing the second flash of light, two neutrinos were emitted along with the positron.) Most of the time the neutrino carries out energy and complicates the analysis of the decay. But in some decays, the neutrino does not play a significant role. Here we will focus on β decays in which the role of the neutrinos can be neglected.

The name *β decay* is historical in origin. When Ernest Rutherford (who later discovered the atomic nucleus) was studying radioactivity in the late 1890s, he noticed that radioactive materials emitted three different kinds of radiation or rays, which he arbitrarily called α (alpha) rays, β (beta) rays and γ (gamma) rays, after the first three letters of the Greek alphabet. Further investigation over the years revealed that α rays were beams of helium nuclei, which are also known as α particles. The β rays turned out to be beams of electrons, and for this reason a nuclear decay in which an electron is emitted is known as a *β decay*. The γ rays turned out to be particles of light which we now call photons.

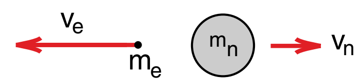
In the 1920s, studies of the β decay process raised serious questions about some fundamental laws of physics. It appeared that in the β decay, energy was sometimes lost. (We will discuss energy in Chapter 9.) In the early 1930s, Wolfgang Pauli proposed that in β decay, two particles were emitted—an electron and an undetectable one which later became known as the neutrino. Pauli's hypothesis was that the missing energy was carried out by the unobservable neutrino. Thirty years later the neutrino was finally detected and Pauli's hypothesis verified.

Some of the time the neutrino created in a β decay carries essentially no energy and has no effect on the behavior of the electron and the nucleus. When this is the case, we have the genuine 2-particle recoil experiment illustrated in Figure (7). This is a recoil experiment in which one of the particles emerges at speeds near the speed of light.

Electron Mass in β Decay

Applying our definition of mass to the β decay process of Figure (7) we have

$$\frac{m_e}{m_n} = \frac{v_n}{v_e} \qquad (5)$$



where m_e and v_e are the mass and recoil speed of the electron, and m_n and v_n of the nucleus. We are assuming that the nucleus was originally at rest before the β decay.

To develop a feeling for the speeds and masses involved in the β decay process, we will analyze two examples of the β decay of a radioactive nucleus. In the first example, which we introduce as an exercise to give you some practice calculating with Equation (5), we can assume that the electron's mass is unchanged and still predict a reasonable speed for the ejected electron. In the second example, the assumption that the electron's mass is unchanged leads to nonsense.

Plutonium 246

We will begin with the decay of a radioactive nucleus called ***Plutonium 246***. This is not a very important nucleus. We have selected it because of the way in which it β decays.

The number 246 appearing in the name tells us the number of protons and neutrons in the nucleus. Protons and neutrons have approximately the same mass m_p which has the value

$$m_p = 1.67 \times 10^{-27}\,\text{kg} \quad \textit{mass of proton} \tag{6}$$

The Plutonium 246 nucleus has a mass 246 times as great, thus

$$\begin{aligned} m_{\text{Plutonium 246}} &= 246 \times m_p \\ &= 4.10 \times 10^{-25}\,\text{kg} \end{aligned} \tag{7}$$

An electron at rest or moving at slow speeds has a mass $(m_e)_0$ given by

$$(m_e)_0 = 9.11 \times 10^{-31}\,\text{kg} \tag{8}$$

This is called the ***rest mass*** of an electron. We have added the subscript zero to remind us that this is the mass of a slowly moving electron, one traveling at speeds much less than the speed of light.

Exercise 2 β Decay of Plutonium 246

A Plutonium 246 nucleus has an average lifetime of just over 11 days, upon which it decays by emitting an electron. If the nucleus is initially at rest, and the decay is one in which the neutrino plays no role, then the nucleus will recoil at the speed

$$v_n = 572\,\frac{\text{meters}}{\text{second}} \quad \begin{array}{l}\textit{recoil speed of}\\ \textit{Plutonium 246}\\ \textit{in a } \beta \textit{ decay}\end{array} \tag{9}$$

This recoil speed is not observed directly, but enough is known about the Plutonium 246 β decay that this number can be accurately calculated. Note that a speed of 572 meters/second is a bit over 1000 miles per hour, the speed of a supersonic jet.

Your exercise is to predict the recoil speed v_e of the electron assuming that the mass of the electron m_e is the same as the mass $(m_e)_0$ of an electron at rest.

Your answer should be

$$v_e = .86\,c \tag{10}$$

where

$$c = 3 \times 10^8\,\frac{\text{meters}}{\text{second}} \tag{11}$$

is the speed of light.

The above exercise, which you should have done by now, shows that we do not get into serious trouble if we assume that the mass of the electron did not change due to the electron's motion. The predicted recoil speed $v_e = .86c$ is a bit too close to the speed of light for comfort, but the calculation does not exhibit any obvious problems. This is not true for the following example.

Protactinium 236

An even more obscure nucleus is Protactinium 236 which has a lifetime of about 12 minutes before it β decays. The Protactinium β decay is, however, much more violent than the Plutonium 246 decay we just discussed. If the Protactinium 236 nucleus is initially at rest, and the neutrino plays no significant role in the decay, then the recoil velocity of the nucleus is

$$v_n = 5170\,\frac{\text{meters}}{\text{second}} \quad \begin{matrix}\textit{recoil speed of}\\ \textit{Protactinium 236}\\ \textit{nucleus}\end{matrix} \tag{12}$$

This is nine times faster than the recoil speed of the Plutonium 246 nucleus.

Exercise 3 Protactinium 236 β decay

Calculate the recoil speed of the electron assuming that the mass of the recoiling electron is the same as the mass of an electron at rest. What is wrong with the answer?

You do not have to work Exercise 3 in detail to see that we get a into trouble if we assume that the mass of the recoiling electron is the same as the mass of an electron at rest. We made this assumption in Exercise 2, and predicted that the electron in the Plutonium 246 β decay emerged at a speed of .86 c. Now a nucleus of about the same mass recoils 9 times faster. If the electron mass is unchanged, it must also recoil 9 times faster, or over seven times the speed of light. This simply does not happen.

Exercise 4 Increase in Electron Mass

Reconsider the Protactinium 236 decay, but this time assume that the electron emerges at essentially the speed of light ($v_e = c$). (This is not a bad approximation, it actually emerges at a speed v = .99 c). Use the definition of mass, Equation (5), to calculate the mass of the recoiling electron. Your answer should be

$$m_e = 6.8 \times 10^{-30}\text{kg} = 7.47 \times (m_e)_0 \tag{13}$$

In Exercise 4, you found that by assuming the electron could not travel faster than the speed of light, the electron mass had increased by a factor of 7.47. The emerging electron is over 7 times as massive as an electron at rest! Instead of emerging at 7 times the speed of light, the electron comes out with 7 times as much mass.

Exercise 5 A Thought Experiment

To illustrate that there is almost no limit to how much the mass of an object can increase, imagine that we perform an experiment where the earth ejects an electron and the earth recoils at a speed of 10 cm/sec. (A β decay of the earth.) Calculate the mass of the emitted electron. By what factor has the electron's mass increased?

THE EINSTEIN MASS FORMULA

A combination of the recoil definition of mass with the observation that nothing can travel faster than the speed of light, leads to the conclusion that the mass of an object must increase as the speed of the object approaches the speed of light. Determining the formula for how mass increases is a more difficult job. It turns out that we do not have enough information at this point in our discussion to derive the mass formula. What we have to add is a new basic law of physics called the ***law of conservation of linear momentum***.

We will discuss the conservation of linear momentum in the next chapter, and in the appendix to that chapter, derive the formula for the increase in mass with velocity. We put the derivation in an appendix because it is somewhat involved. But the answer is very simple, almost what you might guess.

In our discussion of moving clocks in Chapter 1, we saw that the length T' of the astronaut's second increased according to the formula

$$T' = \frac{T}{\sqrt{1-v^2/c^2}} \tag{1-11}$$

where T was the length of one of our seconds. For slowly moving astronauts where $v << c$, we have $T' \approx T$ and the length of the astronaut's seconds is nearly the same as ours. But as the astronaut approaches the speed of light, the number $\sqrt{1-v^2/c^2}$ becomes smaller and smaller, and the astronaut's seconds become longer and longer. If the astronaut goes at the speed of light, $1/\sqrt{1-v^2/c^2}$ becomes infinitely large, the astronaut's seconds become infinitely long, and time stops for the astronaut.

Essentially the same formula applies to the mass of a moving object. If an object has a mass m_0 when at rest or moving slowly as in air cart experiments (we call m_0 the ***rest mass*** of the object), then when the object is moving at a speed v, its mass m is given by the formula

$$\boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}}} \qquad \textit{Einstein mass formula} \tag{14}$$

a result first deduced by Einstein.

Equation (14) has just the properties we want. When the particle is moving slowly as in our air cart recoil experiments, $v << c$, $\sqrt{1-v^2/c^2} \approx 1$ and the mass of the object does not change with speed. But as the speed approaches the speed of light, the $\sqrt{1-v^2/c^2}$ approaches zero, and $m = m_0/\sqrt{1-v^2/c^2}$ increases without bounds. If we could accelerate an object up to the speed of light, it would acquire an infinite mass.

Exercise 6

At what speed does the mass of an object double (i.e., at what speed does $m = 2m_0$?) (Answer: v = .866 c.)

Exercise 7

Electrons emerging from the Stanford Linear Accelerator have a mass 200,000 times greater than their rest mass. What is the speed of these electrons? (The answer is v = .9999999999875 c. Use the approximation formulas discussed in Chapter 1 to work this problem.)

Exercise 8

A car is traveling at a speed of v = 68 miles per hour. (68 miles/hr = 100 ft/second = 10^{-7} ft/nanosecond = 10^{-7} c.) By what factor has its mass increased due to its motion. (Answer: m/m_0 = 1.000000000000005.)

Nature's Speed Limit

When the police try to enforce a 65 mile/hr speed limit, they have a hard job. They have to send out patrol cars to observe the traffic, and chase after speeders. Even with the most careful surveillance, many drivers get away with speeding.

Nature is more clever in enforcing its speed limit c. By having the mass of an object increase as the speed of the object approaches c, it becomes harder and harder to change the speed of the object. If you accelerated an object up to the speed of light, its mass would become infinite, and it would be impossible to increase the particle's speed.

Historically it was noted that massive objects were hard to get moving, but when you got them moving, they were hard to stop. This tendency of a massive object to keep moving at constant velocity was given the name ***inertia***. That is why our recoil definition of mass, which directly measures how hard it is to get an object moving, measures what is called ***inertial mass***. Nature enforces its speed limit c by increasing a particle's inertia to infinity at c, making it impossible to accelerate the particle to higher speeds. Because of this scheme, no one speeds and no police are necessary.

ZERO REST MASS PARTICLES

If you think about it for a while, you may worry that nature's enforcement of its speed limit c is too effective. With the formula $m = m_0/\sqrt{1-v^2/c^2}$, we expect that nothing can reach the speed of light, because it would have an infinite mass, which is impossible.

What is light? It travels at the speed of light. If light consists of a beam of particles, and these particles travel at the speed c, then the formula $m = m_0/\sqrt{1-v^2/c^2}$ suggests that these particles have an infinite mass, which is impossible.

Then perhaps light does not consist of particles, and is therefore exempt from Einstein's formula. Back in Newton's time there was considerable debate over the nature of light. Isaac Newton supported the idea that light consisted of beams of particles. Red light was made up of red particles, green light of green particles, blue light of blue particles, etc. Christian Huygens, a well known Dutch physicist of the time, proposed that light was made up of waves, and that the different colors of light were simply waves with different wavelengths. Huygens developed the theory of wave motion in order to support his point of view. We will discuss Huygens' theory later in the text.

In 1801, about 100 years after the time of Newton and Huygens, Thomas Young performed an experiment that settled the debate they started. With his so called ***two slit experiment*** Young conclusively demonstrated that light was a wave phenomena.

Another century later in 1905, the same year that he published the special theory of relativity, Einstein also published a paper that conclusively demonstrated that light consisted of beams of particles, particles that we now call ***photons***. (Einstein received the Nobel Prize in 1921 for his paper on the nature of light. At that time his special theory of relativity was still too controversial to be awarded the prize.)

Thus by 1905 it was known that light was both a particle and a wave. How this could happen, how to picture something as both a particle and a wave, was not understood until the development of quantum mechanics in the period 1923 through 1925.

Despite the fact that light has a wave nature, it is still made up of beams of particles called photons, and these particles travel at precisely the speed c. If we apply Einstein's mass formula to photons, we get for the photon mass m_{photon}

$$m_{photon} = \left.\frac{m_0}{\sqrt{1-v^2/c^2}}\right|_{v=c} = \frac{m_0}{\sqrt{1-1}} = \frac{m_0}{0} \tag{15}$$

where m_0 is the rest mass of the photon.

At first sight it looks like we are in deep trouble with Equation (15). Division by zero usually leads to a disaster called infinity. There is one exception to this disaster. If the rest mass m_0 of the photon is zero, then we get

$$m_{photon} = \frac{m_0}{0} = \frac{0}{0} \tag{16}$$

The number 0/0 is not a disaster, it is simply undefined. It can be 1 or 2.7, or 6×10^{-23}. It can be any number you want. (How many nothings fit into nothing? As many as you want.) In other words, if the rest mass m_0 of a photon is zero, the Einstein mass formula says nothing about the photon's mass m_{photon}. Photons do have mass, but the Einstein mass formula does not tell us what it is. (Einstein presented a new formula for the photon's mass in his 1905 paper. He found that the photon's mass was proportional to the frequency of the light wave.)

We will study Einstein's theory of photons in detail later in the text. All we need to know now is that light consists of particles called photons, these particles travel ***<u>at</u>*** the speed of light, and these particles have no rest mass. If you stop photons, which you do all the time when light strikes your skin, no particles are left. There is no residue of stopped photons on your skin. All that is left is the heat energy brought in by the light.

A photon is an amazing particle in that it exists only when moving at the speed of light. There is no lapse of time for photons; they cannot become old. (They cannot spontaneously decay like muons, because their half life would be infinite.) There are two different worlds for particles. Particles with rest mass cannot get up to the speed of light, while particles without rest mass travel only at the speed of light.

CHAPTER 6 REVIEW

In this chapter we introduced two different ways to define mass. The first was the recoil definition where we recoiled two air carts from rest and measured their recoil speeds v_A *and* v_B*. The ratio of the cart masses* m_A/m_B *was defined to be equal to the inverse speed ratio* v_B/v_A*. If we defined* m_B *as a standard mass, then any cart recoiled from cart B had a mass*

$$m_A = m_B \frac{v_B}{v_A} \qquad \text{definition of } m_A \qquad (2)$$

The convenient thing to do was to call cart B a unit mass, for example 1 kilogram, so that m_A *would be measured in kilograms.*

An alternate way to define mass is to use a pan balance to compare the weights of two objects. We say that when the balance is level, the objects on the two pans have equal mass. Since this definition depends upon the gravitational force acting on the masses, we call this ***gravitational mass****.*

In contrast, the recoil definition of mass depends on how hard it is to shove the two carts apart. This introduces the historical concept of inertia, and we can say that the recoil definition measures the ***inertial mass*** *of an object. Very careful experiments have demonstrated that inertial and gravitational mass are equal.*

Mass of a Moving Object

One of the main results of studying the recoil of two carts was that the recoil velocity ratio v_B/v_A *did not change as we used stronger springs and the carts came out faster. Our conclusion was that the mass of a cart did not depend on its velocity. This was true at least for the rather slow speeds we can observe for carts on air tracks.*

The remainder of the chapter deals with what happens to particles moving at speeds near the speed of light. We see that, when you combine the recoil definition of mass with the results of special relativity, the mass of an object must increase as its speed approaches the speed of light.

Einstein Mass Formula

Einstein derived a formula for how the mass of an object increased as its speed v approaches the speed of light. The result is

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \qquad (14)$$

where m_0 *is the so-called* ***rest mass****, the mass of the particle when recoiled at low speeds.*

On the highway, speed limits are inforced by having patrol cars with policemen to catch speeders. Nature does not need policemen to enforce nature's speed limit c. When an object's speed approaches c, its mass becomes infinite, and the object cannot go faster.

Particles of light are an exception. Light particles, called ***photons****, travel* ***at*** *the speed of light, so for them*

$$\sqrt{1-v^2/c^2} = \sqrt{1-c^2/c^2} = \sqrt{0} = 0$$

and $m = m_0/0$*. Division by zero usually gives infinity. But if the photon's rest mass* m_0 *is zero, then Einstein's formula gives*

$$m_{photon} = \frac{m_0}{0} = \frac{0}{0} = \text{anything} \qquad (16)$$

Since 0/0 can be anything (how many nothings can you fit into nothing), another formula has to determine the photon's mass. Einstein came up with that formula the same year he came up with special relativity. His formula is the ***photoelectric formula*** *which we will discuss in Chapter 26 on photons.*

CHAPTER EXERCISES

Exercise 1 0n page 4

Suppose that you were given a rod of brass and a hacksaw. Describe in detail how you would construct a set of standard masses.

Exercise 2 0n page 8

Analyze the β decay of Plutonium 246.

Exercise 3 0n page 9

Analyze the Protactinium 236 β decay.

Exercise 4 0n page 9

Calculate the mass of the recoiling electron in the Protactinium 236 β decay.

Exercise 5 0n page 9

A thought experiment where the earth ejects an electron and the earth recoils at a speed of 10 cm/sec.

Exercise 6 0n page 10

At what speed does the mass of an object double?

Exercise 7 0n page 10

Electrons emerging from the Stanford Linear Accelerator have a mass 200,000 times greater than their rest mass. What is the speed of these electrons?

Exercise 8 0n page 10

A car is traveling at a speed of v = 68 miles per hour (100 ft/sec). By what factor has its mass increased due to its motion.

REVIEW QUESTIONS

9) In the definition of mass, why is it necessary to have a standard mass?

10) How does nature enforce nature's speed limit c?

11) We think that it takes 8 minutes for a photon (particle of light) to travel from the sun to the earth? How long do the photons think it takes?

12) Why can't we imagine photons thinking?

13) How do photons avoid having an infinite mass?