Chapter 30
Faraday's Law

In this chapter we will discuss one of the more remarkable, and in terms of practical impact, important laws of physics – Faraday's law. This law explains the operation of the air cart speed detector we have used in air track experiments, the operation of AC voltage generators that supply most of the electrical power in the world, and transformers and inductors which are important components in the electronic circuits in radio and television sets.

In one form, Faraday's law deals with the line integral $\oint \vec{E}\cdot d\vec{\ell}$ *of an electric field around a closed path. As an introduction we will begin with a discussion of this line integral for electric fields produced by static charges. (Nothing very interesting happens there.) Then we will analyze an experiment that is similar to our air cart speed detector to see why we get a voltage proportional to the speed of the air cart. Applying the principle of relativity to our speed detector, i.e., riding along with the air cart gives us an entirely new picture of the behavior of electric fields, a behavior that is best expressed in terms of the line integral* $\oint \vec{E}\cdot d\vec{\ell}$. *After a discussion of this behavior, we will go through some practical applications of Faraday's law.*

ELECTRIC FIELD OF STATIC CHARGES

In this somewhat formal section, we show that $\oint \vec{E}\cdot d\vec{\ell} = 0$ for the electric field of static charges. With this as a background, we are in a better position to appreciate an experiment in which $\oint \vec{E}\cdot d\vec{\ell}$ is not zero.

In Figure (1), we have sketched a closed path through the electric field $\vec{E}$ of a point charge, and wish to calculate the line integral $\oint \vec{E}\cdot d\vec{\ell}$ for this path. To simplify the calculation, we have made the path out of arc and radial sections. But as in our discussion of Figure 29-13, we can get arbitrarily close to any path using arc and radial sections, thus what we learn from the path of Figure (1) should apply to a general path.

Because the electric field is radial, $\vec{E}$ is perpendicular to $d\vec{\ell}$ and $\vec{E}\cdot d\vec{\ell}$ is zero on the arc sections. On the radial sections, for every step out where $\vec{E}\cdot d\vec{r}$ is positive there is an exactly corresponding step back where $\vec{E}\cdot d\vec{r}$ is negative. Because we come back to the starting point, we take the same steps back as we took out, all the radial $\vec{E}\cdot d\vec{r}$ cancel and we are left with $\oint \vec{E}\cdot d\vec{\ell} = 0$ for the electric field of a point charge.

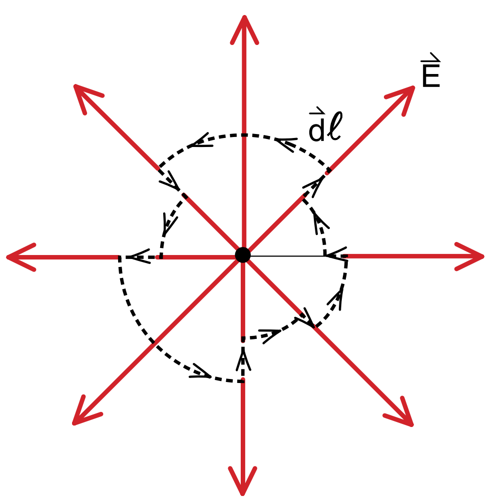


Figure 1
Closed path through the electric field of a point charge. The product $\vec{E}\cdot d\vec{\ell}$ is zero on the arc sections, and the path goes out just as much as it comes in on the radial sections. As a result $\oint \vec{E}\cdot d\vec{\ell} = 0$ when we integrate around the entire path.

Now consider the distribution of fixed point charges shown in Figure (2). Let $\vec{E}_1$ be the field of Q_1, $\vec{E}_2$ of Q_2, etc. Because an electric field is the force on a unit test charge, and because forces add as vectors, the total electric field $\vec{E}$ at any point is the vector sum of the individual fields at that point

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 \tag{1}$$

We can now use Equation (1) to calculate $\oint \vec{E}\cdot d\vec{\ell}$ around the closed path in Figure (2). The result is

$$\begin{aligned}\oint \vec{E}\cdot d\vec{\ell} &= \oint \left(\vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_5\right)\cdot d\vec{\ell} \\ &= \oint \vec{E}_1\cdot d\vec{\ell} + \cdots + \oint \vec{E}_5\cdot d\vec{\ell}\end{aligned} \tag{2}$$

But $\oint \vec{E}_1\cdot d\vec{\ell} = 0$ since $\vec{E}_1$ is the field of a point charge, and the same is true for $\vec{E}_2 \ \ldots \ \vec{E}_5$. Thus the right side of Equation (2) is zero and we have

$$\oint \vec{E}\cdot d\vec{\ell} = 0 \quad \left\{ \begin{array}{l}\text{for the field } \vec{E} \text{ of} \\ \text{any distribution of} \\ \text{static charges}\end{array}\right. \tag{3}$$

Equation (3) applies to any distribution of static charges, a point charge, a line charge, and static charges on conductors and in capacitors.

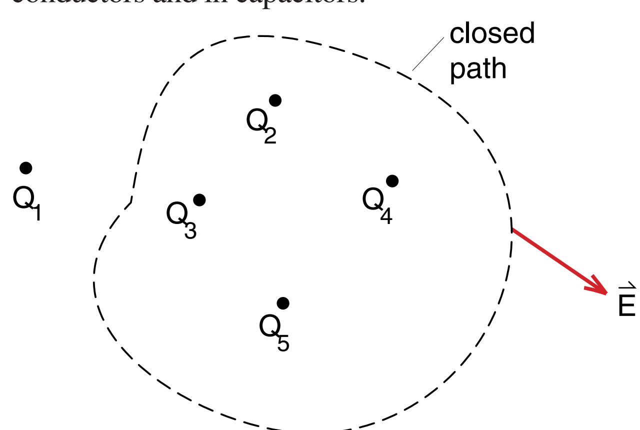


Figure 2
Closed path in a region of a distribution of point charge. Since $\oint \vec{E}\cdot d\vec{\ell} = 0$ is zero for the field of each point charge alone, it must also be zero for the total field $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5$

A MAGNETIC FORCE EXPERIMENT

Figures (3a,b) are two views of an experiment designed to test for the magnetic force on the conduction electrons in a moving copper wire. We have a wire loop with a gap and the loop is being pulled out of a magnet. At this instant only the end of the loop, the end opposite the gap, is in the magnetic field. It will soon leave the field since it is being pulled out at a velocity $\vec{v}$ as shown.

In our earlier discussions we saw that a copper atom has two loosely bound conduction electrons that are free to flow from one atom to another in a copper wire. These conduction electrons form a negatively charged electric fluid that flows in a wire much like water in a pipe.

Because of the gap we inserted in the wire loop of Figure (3), the conduction electrons in this loop cannot flow. If we move the loop, the conduction electrons must move with the wire. That means that the conduction electrons have a velocity $\vec{v}$ to the right as shown, perpendicular to the magnetic field which is directed into the page. Thus we expect that there should be a magnetic force

$$\vec{F}_{mag} = -e\vec{v}\times\vec{B} \qquad (4)$$

acting on the electrons. This force will be directed down as shown in Figure (3b).

Since the gap in the loop does not allow the conduction electrons to flow along the wire, how are we going to detect the magnetic force on them? There is no net force on the wire because the magnetic field exerts an equal and opposite force on the positive copper ions in the wire.

Our conjecture is that this magnetic force on the conduction electrons would act much like the gravitational force on the water molecules in a static column of water. The pressure at the bottom of the column is higher than the pressure at the top due to the gravitational force. Perhaps the pressure of the negatively

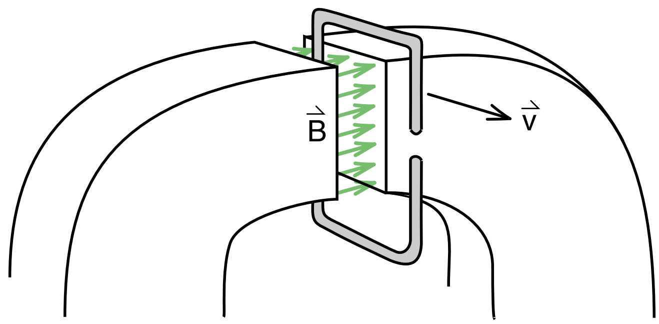


Figure 3a
Wire loop moving through magnetic field of iron magnet.

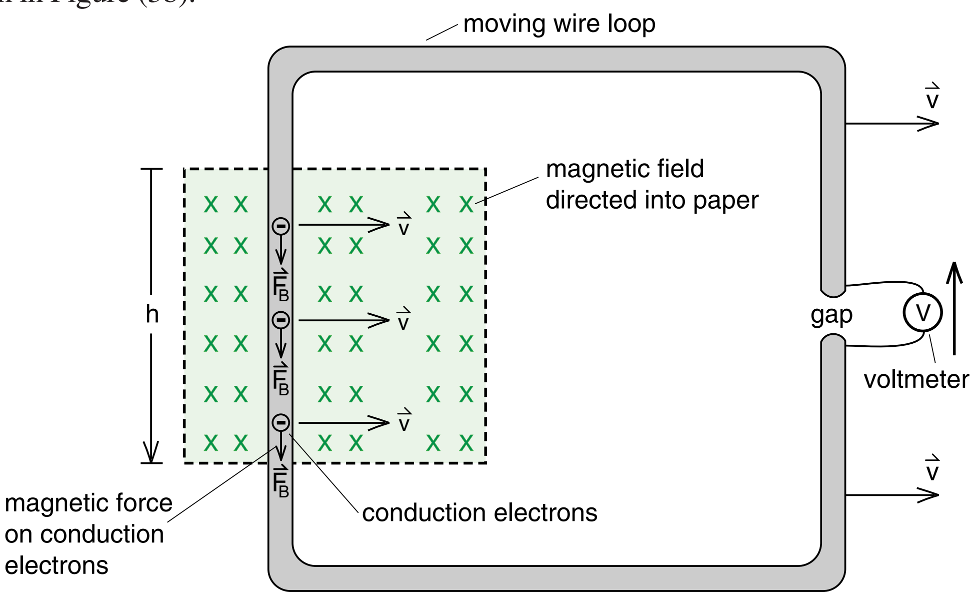


Figure 3b
When you pull a wire loop through a magnetic field, the electrons, moving at a velocity $\vec{v}$ with the wire, feel a magnetic force $\vec{F}_B = (-e)\vec{v}\times\vec{B}$ if they are in the field. This force raises the pressure of the electron fluid on the bottom of the loop and reduces it on the top, creating a voltage V across the gap. The arrow next to the voltmeter indicates a voltage rise for positive charge, which is a voltage drop for negative charge.

charged electric fluid is higher at the bottom of the loop than the top due to the magnetic force.

To find out if this is true, we use an electrical pressure gauge, which is a voltmeter. A correctly designed voltmeter measures an electrical pressure drop without allowing any current to flow. Thus we can place the voltmeter across the gap and still not let the conduction electrons flow in the loop.

If our conjecture is right, we should see a voltage reading while the magnetic force is acting. Explicitly there should be a voltage reading while the wire is moving and one end of the loop is in the magnetic field as shown. The voltage should go to zero as soon as the wire leaves the magnetic field. If we reverse the direction of motion of the loop, the velocity $\vec{v}$ of the conduction electrons is reversed, the magnetic force $-e\vec{v} \times \vec{B}$ should also be reversed, and thus the sign of the voltage on the voltmeter should reverse. If we oscillate the wire back and forth, keeping one end in the magnetic field, we should get an oscillating voltage reading on the meter.

The wonderful thing about this experiment is that all these predictions work precisely as described. There are further simple tests like moving the loop faster to get a stronger magnetic force and therefore a bigger voltage reading. Or stopping the wire in the middle of the magnetic field and getting no voltage reading. They all work!

The next step is to calculate the magnitude of the voltage reading we expect to see. As you follow this calculation, do not worry about the sign of the voltage V because many sign conventions (right hand rules, positive charge, etc.) are involved. Instead concentrate on the basic physical ideas. (In the laboratory, the sign of the voltage V you read on a voltmeter depends on how you attached the leads of the voltmeter to the apparatus. If you wish to change the sign of the voltage reading, you can reverse the leads.)

Since voltage has the dimensions of the potential energy of a unit test charge, the magnitude of the voltage in Figure (3) should be the strength of the force on a unit test charge, $(-e)\,\vec{v} \times \vec{B}$ with $(-e)$ replaced by 1, times the height h over which the force acts. This height h is the height of the magnetic field region in Figure (3). Since v and B are perpendicular, $\left|\vec{v} \times \vec{B}\right| = vB$ and we expect the voltage V to be given by

$$V = \begin{pmatrix} \text{force on unit} \\ \text{test charge} \end{pmatrix} \times \begin{pmatrix} \text{distance over} \\ \text{which force acts} \end{pmatrix}$$

$$\boxed{V = vB \times h} \qquad \begin{matrix} \textit{voltage V on loop} \\ \textit{moving at speed} \\ \textit{v through field B} \end{matrix} \qquad (5)$$

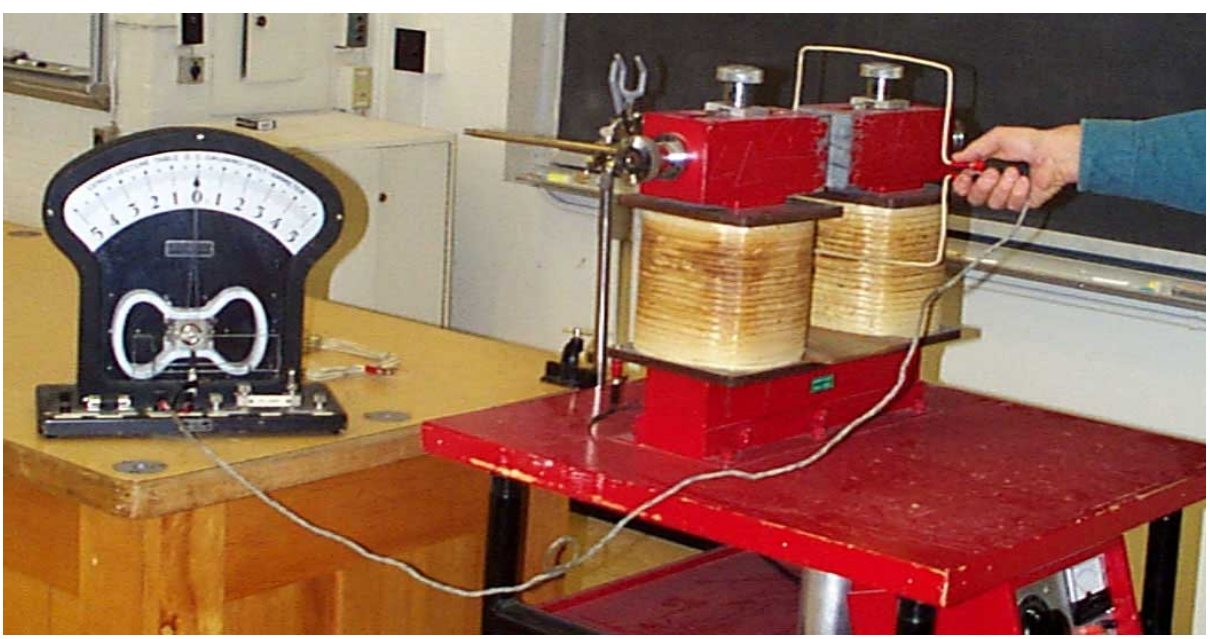


Figure 3c
Pulling the coil out of the magnet

AIR CART SPEED DETECTOR

The air cart velocity detector we have previously discussed, provides a direct verification of Equation (5). The only significant difference between the air cart speed detector and the loop in Figure (3) is that the speed detector coil has a number of turns (usually 10). In order to see the effect of having more than one turn in the coil, we show a two turn coil being pulled out of a magnetic field in Figure (4).

Figure (4) is beginning to look like a plumbing diagram for a house. To analyze the diagram, let us start at Position (1) at the top of the voltmeter and follow the wire all the way around until we get to Position (6) at the bottom end of the voltmeter. When we get to Position (2), we enter a region from (2) to (3) where the magnetic force is increasing the electron fluid pressure by an amount vBh, as in Figure (3).

Now instead of going directly to the voltmeter as in Figure (3), we go around until we get to Position (4) where we enter another region, from (4) to (5), where the magnetic force is increasing the fluid pressure. We get another increase of vBh, and then go to Position (6) at the bottom of the voltmeter. In Figure (4) we have two voltage rises as we go around the two loops, and we should get twice the reading on the voltmeter.

$$V = 2vBh \qquad \textit{voltage reading for 2 loops}$$

It is an easy abstraction to see that if our coil had N turns, the voltage rise would be N times as great, or

$$\boxed{V = NvBh} \qquad \textit{voltage on an N turn coil being pulled out of a magnetic field} \tag{6}$$

Adding more turns is an easy way to increase or amplify the voltage.

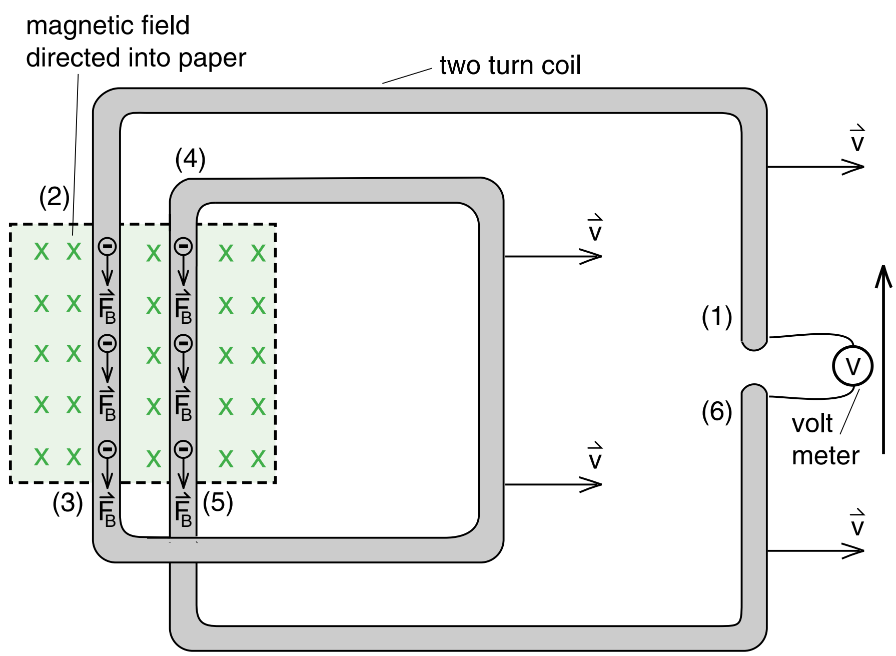


Figure 4
A two turn loop being pulled through a magnetic field. With two turns we have twice as much force pushing the electric fluid toward the bottom of the gap giving twice the voltage V.

The setup for the air cart speed detector is shown in Figure (6). A multi turn coil, etched on a circuit board as shown in Figure (5), is mounted as a sail on top of an air cart. Suspended over the air cart are two angle iron bars with magnets set across the top as shown. This produces a reasonably uniform magnetic field that goes across from one bar to the other as seen in the end view of Figure (6).

In Figure (7), we show the experiment of letting the cart travel at constant speed through the velocity detector. In the initial position (a), the coil has not yet reached the magnetic field and the voltage on the coil is zero, as indicated in the voltage curve at the bottom of the figure.

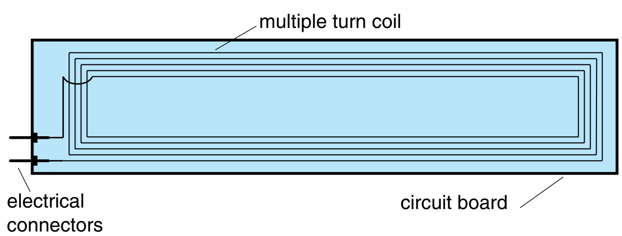


Figure 5
The multi turn coil that rides on the air cart. (Only 5 turns are shown.)

The situation most closely corresponding to Figure (4) is position (d) where the coil is leaving the magnet. According to Equation (6), the voltage at this point should be given by $V = NvBh$, where $N = 10$ for our 10 turn coil, v is the speed of the carts, B is the strength of the magnetic field between the angle iron bars, and h is the average height of the coils. (Since the coils are drawn on a circuit board the outer loop has the greatest height h and the inner loop the least.) The first time you use this apparatus, you can directly measure V, N, v and h and use Equation (6) to determine the magnetic field strength B. After that, you know the constants N, B and h, and Equation (6) written as

$$v = V \times \left(\frac{1}{NBh} \right) \tag{6a}$$

gives you the cart's speed in terms of the measured voltage V. Equation (6a) explains why the apparatus acts as a speed detector.

Let us look at the voltage readings for the other cart positions. The zero readings at Positions (a) and (e) are easily understood. None of the coil is in the magnetic field and therefore there is no magnetic force or voltage.

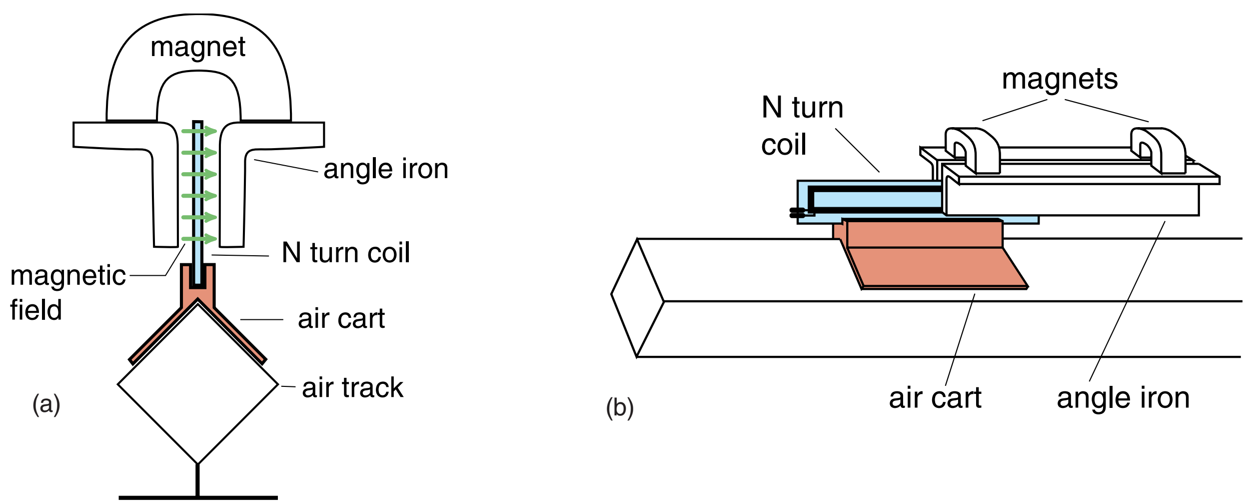


Figure 6
The Faraday velocity detector. The apparatus is reasonably easy to build. We first constructed a 10 turn coil by etching the turns of the coil on a circuit board. This was much better than winding a coil, for a wound coil tends to have wrinkles that produce bumps in the data. Light electrical leads, not shown, go directly from the coil to the oscilloscope. The coil is mounted on top of an air cart and moves through a magnetic field produced by two pieces of angle iron with magnets on top as shown. Essentially we have reproduced the setup shown in Figures 3 and 4, but with the coil mounted on an air cart. As long as the coil remains with one end in the magnetic field and the other outside, as shown in (b), there will be a voltage on the leads to the coil that is proportional to the velocity of the cart.

Figure 6c
Velocity detector apparatus. The magnetic field goes across, between the two pieces of angle iron. The coil, mounted on a circuit board, is entering the magnetic field.

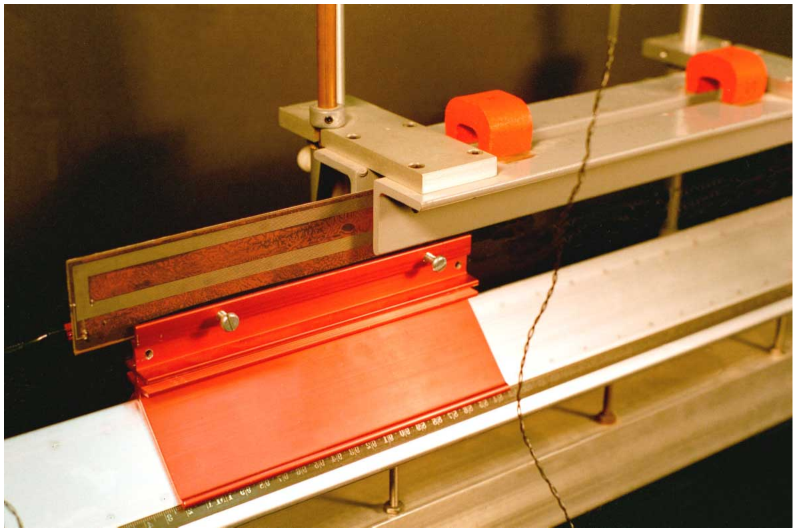
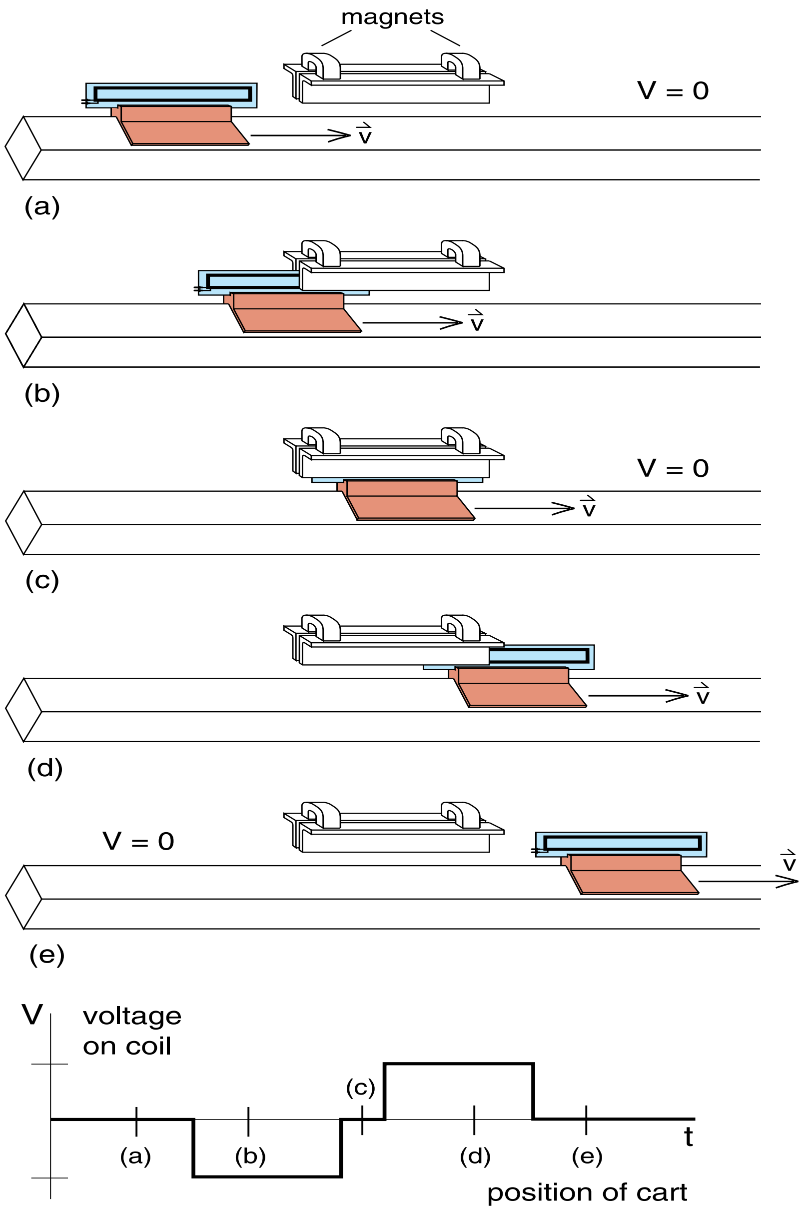


Figure 7
Voltage on the coil as it moves at constant speed through the magnetic field. At position (a) the coil has not yet reached the field and there is no voltage. At position (b) one end of the cart is in the field, the other outside, and we get a voltage proportional to the speed of the cart. At (c) there is no voltage because both ends of the cart are in the magnetic field and the magnetic force on the two ends cancel. (There is no change of magnetic flux at this point.) At (d), the other end alone is inside the field, and we get the opposite voltage from the one we had at (b). (Due to the thickness of the coil and fringing of the magnetic field, the voltage rises and falls will be somewhat rounded.)

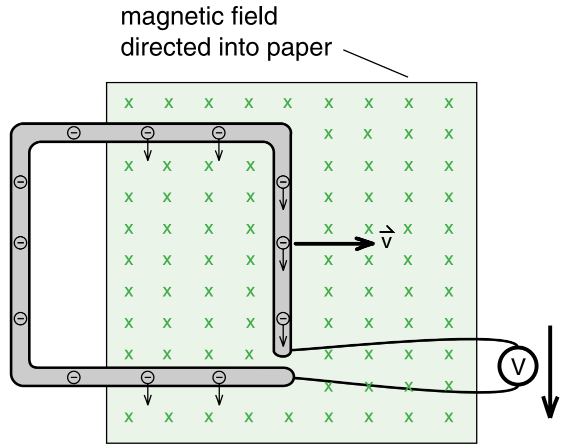


We need a closer look to understand the changes in voltage, when all or part of the coil is inside the magnetic field. This situation, for a one turn coil, is illustrated in Figure (8). For easier interpretation we have moved the gap and voltmeter to the bottom of the coil as shown. It turns out that it does not matter where the gap is located, we get the same voltage reading. We have also labeled the figures (b), (c), and (d) to correspond to the positions of the air cart in Figure (7).

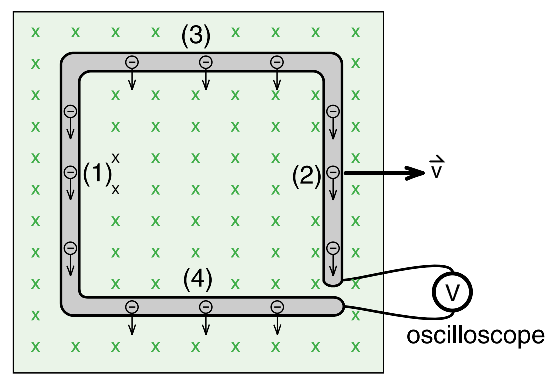
In Figure (8c) where both ends of the coil are in the magnetic field, the conduction electrons are being pulled down in both ends and the fluid is balanced. The electron fluid would not flow in either direction if the gap were closed, thus there is no pressure across the gap and no voltage reading. In contrast, in Figure (8d) where only the left end of the coil is in the magnetic field, the magnetic force on the left side would cause the conduction electrons to flow counterclockwise around the loop if it were not for the gap. There must be an electric pressure or voltage drop across the gap to prevent the counterclockwise flow. This voltage drop is what we measure by the voltmeter.

In Figure (8b), where the coil is entering the magnetic field, the magnetic force on the right side of the coil would try to cause a clockwise flow of the conduction electrons. We should get a pressure or voltage opposite to Figure (8d) where the coil is leaving. This reversal in voltage is seen in the air cart experiment of Figure (7), as the cart travels from (b) to (d).

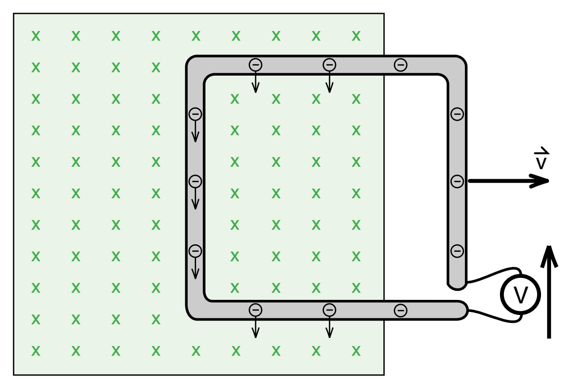
Note that in Figure (8), where the horizontal sections of the coil are also in the magnetic field, the magnetic force is across rather than along the wire in these sections. This is like the gravitational force on the fluid in a horizontal section of pipe. It does not produce any pressure drops.



(b) coil entering magnetic field



(c) coil completely in magnetic field



(d) coil leaving magnetic field

Figure 8
When the coil is completely in the magnetic field, the magnetic force on the electrons in the left hand leg (1) is balanced by the force on the electrons in the right hand leg (2), and there is no net pressure or voltage across the gap. When the coil is part way out, there is a voltage across the gap which balances the magnetic force on the electrons. The sign of the voltage depends upon which leg is in the magnetic field.

A RELATIVITY EXPERIMENT

Now that we have seen, from Figure (7), extensive experimental evidence for the magnetic force on the conduction electrons in a wire, let us go back to Figure (3) where we first considered these forces, and slightly modify the experiment. Instead of pulling the coil out of the magnet, let us pull the magnet away from the coil as shown in Figure (9b).

In Figure (9a) we have redrawn Figure (3), and added a stick figure to represent a student who happens to be walking by the apparatus at the same speed that we are pulling the coil out of the magnet. To this moving observer, the coil is at rest and she sees the magnet moving to the left as shown in (9b). In other words, pulling the magnet away from the coil is precisely the same experiment as pulling the coil from the magnet, except it is viewed by a moving observer.

The problem that the moving observer faces in Figure (9b) is that, to her, the electrons in the coil are at rest. For her the electron speed is v = 0 and the magnetic force $\vec{F}_B$, given by

$$\vec{F}_B = (-e)\vec{v} \times \vec{B} = 0 \quad (\text{for Figure 9b}) \tag{7}$$

is zero! Without a magnetic force to create the pressure in the electrical fluid in the wire, she might predict that there would be no voltage reading in the voltmeter.

But there is a voltage reading on the voltmeter! We have used this voltage to build our air cart velocity detector. If the voltmeter had a digital readout, for example, then it is clear that everyone would read the same number no matter how they were moving, whether they were like us moving with the magnet (9a), or like her moving with the coil (9b). In other words, she has to find some way to explain the voltage reading that she must see.

The answer she needs lies in the Lorentz force law that we discussed in Chapter 28. This law tells us the total electromagnetic force on a charge q due to either electric or magnetic fields, or both. We wrote the law in the form

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{28-20}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields acting on the charge.

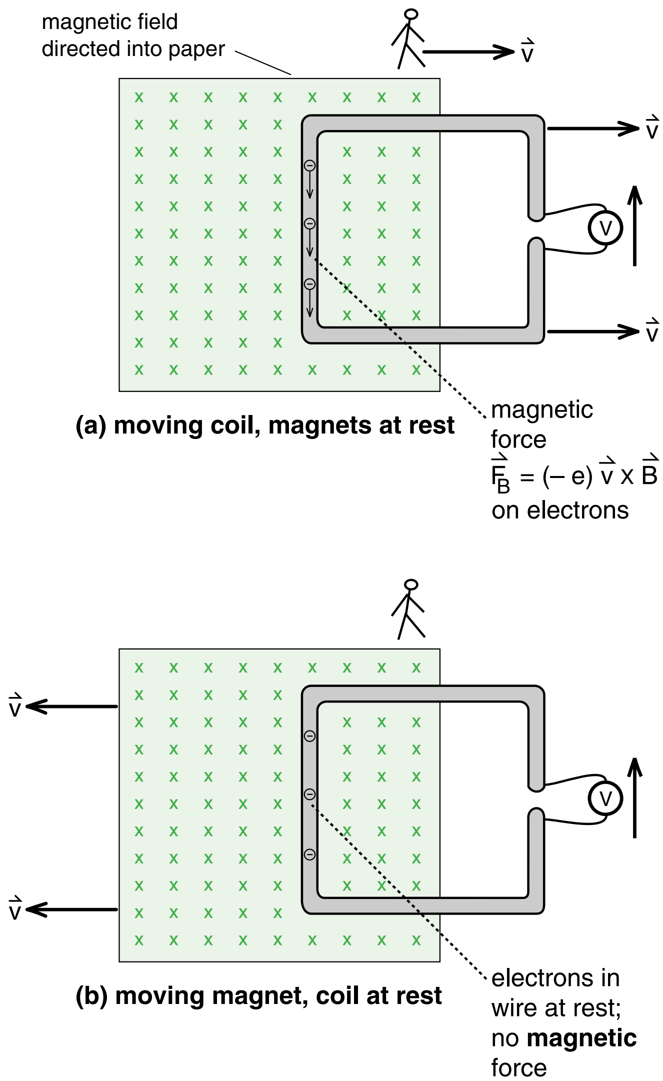


Figure 9
The only difference between (a) and (b) is the point of view of the observer. In (a) we see a magnetic force $\vec{F}_B = (-e)\,\vec{v} \times \vec{B}$ because the electrons are moving at a speed v through a magnetic field $\vec{B}$. To the observer in (b), the magnet is moving, not the electrons. Since the electrons are at rest, there is no magnetic force on them. Yet the voltmeter reading is the same from both points of view.

Let us propose that the Lorentz force law is generally correct even if we change coordinate systems. In Figure (9a) where we explained everything in terms of a magnetic force on the conduction electrons, there was apparently no electric field and the Lorentz force law gave

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$$= (-e)\vec{v}\times\vec{B} \qquad \begin{pmatrix} \textit{in Figure 9a,} \\ \vec{E}=0 \end{pmatrix} \tag{8a}$$

In Figure (9b), where $\vec{v} = 0$, we have

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$$= (-e)\vec{E} \qquad \begin{pmatrix} \textit{in Figure 9b,} \\ \vec{v}=0 \end{pmatrix} \tag{8b}$$

In other words, we will assume that the magnetic force of Figure (9a) has become an electric force in Figure (9b). Equating the two forces gives

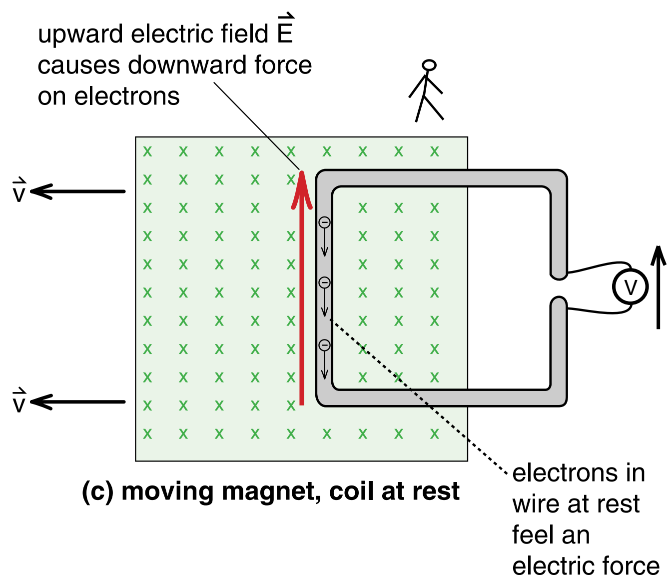
$$\vec{E}\begin{pmatrix} \textit{That should be} \\ \textit{in Figure 9b} \end{pmatrix} = \vec{v}\times\vec{B}\begin{pmatrix} \textit{From} \\ \textit{Figure 9a} \end{pmatrix} \tag{9}$$

In Figure (9c) we have redrawn Figure (9b) showing an electric field causing the force on the electrons. Because the electrons have a negative charge, the electric field must point up in order to cause a downward force.

That the magnetic force of Figure (9a) becomes an electric force in Figure (9c) should not be a completely surprising result. In our derivation of the magnetic force law, we also saw that an electric force from one point of view was a magnetic force from another point of view. The Lorentz force law, which includes both electric and magnetic forces, has the great advantage that it gives the correct electromagnetic force from any point of view.

Exercise 1

Equation (9) equates $\vec{E}$ in Figure (9c) with $\vec{v}\times\vec{B}$ in Figure (9a). Show that $\vec{E}$ and $\vec{v}\times\vec{B}$ point in the same direction.



(c) moving magnet, coil at rest

Figure 9c
From the point of view that the coil is at rest, the downward force on the electrons in the coil must be produced by an upward directed electric field.

FARADAY'S LAW

An experiment whose results may be surprising, is shown in Figure (10). Here we have a magnetic field produced by an electromagnet so that we can turn $\vec{B}$ on and off. We have a wire loop that is large enough to surround but not lie in the magnetic field, so that $\vec{B} = 0$ all along the wire. Again we have a gap and a voltmeter to measure any forces that might be exerted on the conduction electrons in the wire.

We have seen that if we pull the wire out of the magnet, Figure (9a), we will get a voltage reading while the loop is leaving the magnetic field. We have also seen, Figure (9c), that we get a voltage reading if the magnetic field is pulled out of the loop. In both cases we started with a magnetic field through the loop, ended up with no magnetic field through the loop, and got a reading on the voltmeter while the amount of magnetic field through the loop was decreasing.

Now what we are going to do in Figure (10) is simply shut off the electromagnet. Initially we have a magnetic field through the loop, finally no field through the loop. It may or may not be a surprise, but ***when we shut off the magnetic field, we also get a voltage reading***. We get a voltage reading if we pull the loop out of the field, the field out of the loop, or shut off the field. We are seeing that we ***get a voltage reading whenever we change the amount of magnetic field, the flux of magnetic field, through the loop***.

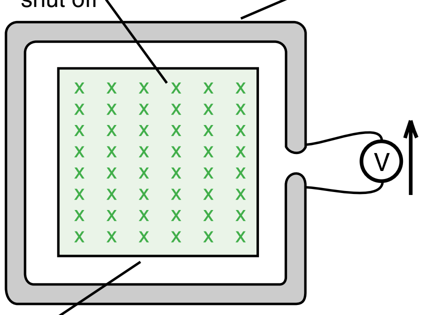


Figure 10
Here we have a large coil that lies completely outside the magnetic field. Thus there is no magnetic force on any of the electrons in the coil wire. Yet when we turn the magnet on or off, we get a reading in the volt meter.

Magnetic Flux

In our discussion of velocity fields and electric fields, we used the concept of the flux of a field. For the velocity field, the flux Φ_v of water was the volume of water flowing per second past some perpendicular area $A_\perp$. For a uniform stream moving at a speed v, the flux was $\Phi_v = vA_\perp$. For the electric field, the formula for flux was $\Phi_E = EA_\perp$.

In Figures (9 and 10), we have a magnetic field that "flows" through a wire loop. Following the same convention that we used for velocity and electric fields, we will define the magnetic flux Φ_B as the strength of the field $\vec{B}$ times the perpendicular area $A_\perp$ through which the field is flowing

$$\boxed{\Phi_B = BA_\perp} \qquad \begin{array}{c}\text{Definition of}\\ \text{magnetic flux}\end{array} \qquad (10)$$

In both figures (9) and (10), the flux Φ_B through the wire loop is decreasing. In Figure (9), Φ_B decreases because the perpendicular area $A_\perp$ is decreasing as the loop and the magnet move apart. In Figure (10), the flux Φ_B is decreasing because $\vec{B}$ is being shut off. The important observation is that whenever the flux Φ_B through the loop decreases, whatever the reason for the change may be, we get a voltage reading V on the voltmeter.

One Form of Faraday's Law

The precise relationship between the ***voltage*** and the ***change in the magnetic flux*** through the loop is found from our analysis of Figure (9) where the loop and the magnet were pulled apart. We got a voltage given by Equation (5) as

$$V = vBh \tag{5}$$

Let us apply Equation (5) to the case where the magnet is being pulled out of the loop as shown in Figure (11). In a time dt, the magnet moves to the left a distance dx given by

$$dx = vdt \tag{11}$$

and the area of magnetic field that has left the loop, shown by the cross hatched band in Figure (11), is

$$dA = hdx = \left\{ \begin{array}{l} \text{area of magnetic} \\ \text{field that has} \\ \text{left the loop} \end{array} \right. \tag{12}$$

This decrease in area causes a decrease in the magnetic flux $\Phi_B = BA_\perp$ through the loop. The change in flux $d\Phi_B$ is given by

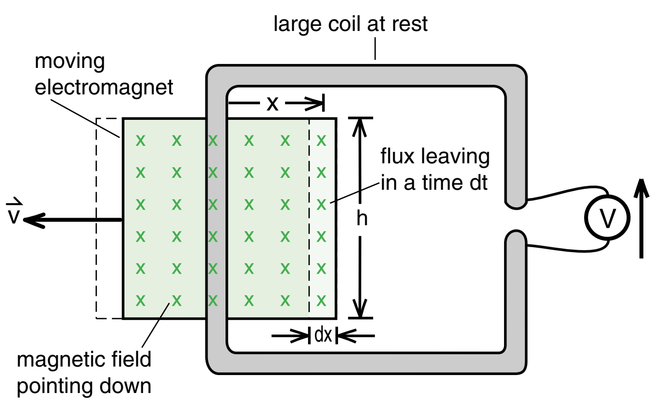


Figure 11
As the magnet and the coil move away from each other, the amount of magnetic flux through the coil decreases. When the magnet has moved a distance dx, the decrease in area is hdx, and the magnetic flux decreases by B×hdx.

$$\begin{aligned} d\Phi_B &= -BdA = -Bhdx \\ &= -Bhvdt \end{aligned} \tag{13}$$

where the – sign indicates a reduction in flux, and we used Equation (11) to replace dx by vdt.

Dividing both sides of Equation (13) by dt gives

$$\frac{d\Phi_B}{dt} = -Bhv \tag{14}$$

But Bhv is just our voltmeter reading. Thus we get the surprisingly simple formula

$$\boxed{V = -\frac{d\Phi_B}{dt}} \quad \textit{One form of Faraday's law} \tag{15}$$

Equation (15) is one form of Faraday's law.

Equation (15) has a generality that goes beyond our original analysis of the magnetic force on the conduction electrons. It makes no statement about what causes the magnetic flux to change. We can pull the loop out of the field as in Figure (9a), the field out of the loop as in Figure (9b), or shut the field off as in Figure (10). In all three cases Equation (15) predicts that we should see a voltage, and we do.

If we have a coil with more than one turn, as we had back in Figure (4), and put a voltmeter across the ends of the coil, then we get N times the voltage, and Equation (15) becomes

$$V = N\left(-\frac{d\Phi_B}{dt}\right) \quad \textit{for a coil with N turns} \tag{15a}$$

provided $d\Phi_B/dt$ is the rate of change of magnetic flux in each loop of the coil.

Exercise 2

Go back to Figure (7) and explain the voltage plot in terms of the ***rate of change of the flux of magnetic field*** through the coil riding on top of the air cart.

A Circular Electric Field

In Figure (10), where we shut the magnet off and got a voltage reading on the voltmeter, there must have been some force on the electrons in the wire to produce the voltage. Since there was no magnetic field out at the wire, the force must have been produced by an electric field. We already have a hint of what that electric field looks like from Figure (9c). In that figure, we saw that the moving magnetic field created an upwardly directed electric field acting on the electrons on the left side of the wire loop.

To figure out the shape of the electric field produced when we shut off the magnet, consider Figure (12), where we have a circular magnet and a circular loop of wire . We chose this geometry so that the problem would have circular symmetry (except at the gap in the loop).

To produce the same kind of voltage V that we have seen in the previous experiments, the electric field at the wire must be directed up on the left hand side, as it was in Figure (9c). But because of the circular symmetry of the setup in Figure (12), the upwardly directed electric field on the left side, which is parallel to the wire, must remain parallel to the wire as we go around the wire loop. In other words, the only way we can have an upwardly directed electric field acting on the electrons on the left side of the loop, and maintain circular symmetry, is to have the electric field go in a circle all the way around the loop as shown in Figure (12).

We can determine the strength of this circular electric field, by figuring out how strong an electric field must act on the electrons in the wire, in order to produce the voltage V across the gap. We then use Equation (15) to relate this voltage to the rate of change of the magnetic flux through the loop.

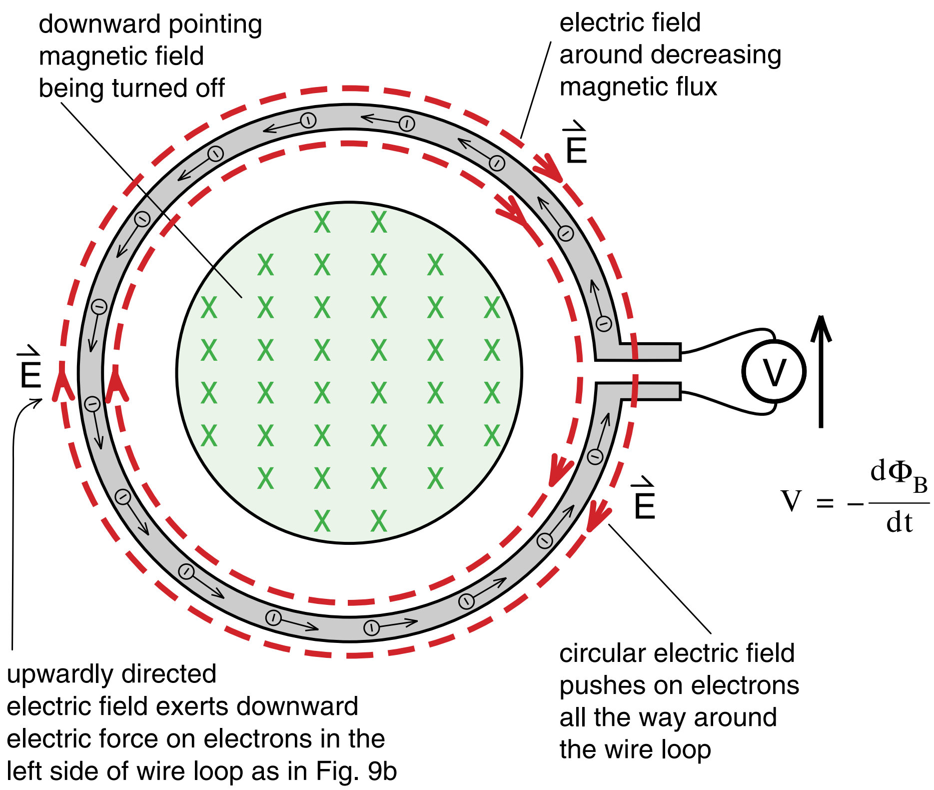


Figure 12
When the magnetic field in the magnet is turned off, a circular electric field is generated. This electric field exerts a force on the electrons in the wire, creating a pressure in the electric fluid that is recorded as a voltage pulse by the voltmeter.

Recall that the definition of electric voltage used in deriving Equation (5) was

$$V = \left(\begin{array}{l}\text{force on unit}\\ \text{test charge}\end{array}\right) \times \left(\begin{array}{l}\text{distance over}\\ \text{which force acts}\end{array}\right)$$

For Figure (12), the force on a unit test charge is the electric field $\vec{E}$, and this force acts over the full circumference $2\pi r$ of the wire loop. Thus the voltage V across the gap is

$$V = E \times 2\pi r$$

Equating this voltage to the rate of change of magnetic flux through the wire loop gives

$$V = E \times 2\pi r = -\frac{d\Phi_B}{dt} \qquad (16)$$

Equation (16) tells us that the faster the magnetic field dies, i.e. the greater $d\Phi_B/dt$, the stronger the electric field $\vec{E}$ produced.

Line Integral of $\vec{E}$ around a Closed Path

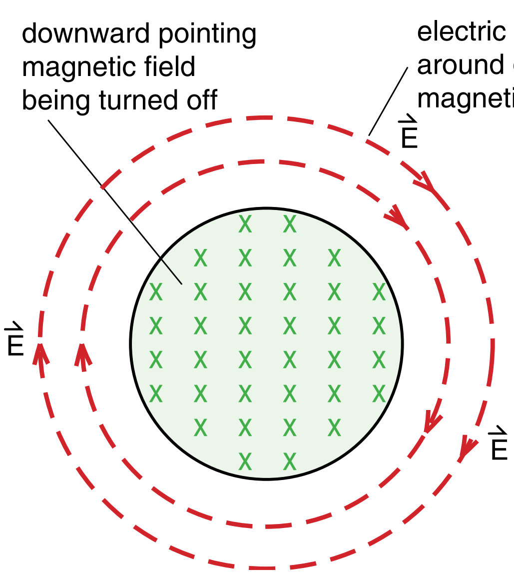
In Figure (13) we have removed the wire loop and volt meter from Figure (12) so that we can focus our attention on the circular electric field produced by the decreasing magnetic flux. This is not the first time we have encountered a circular field. The velocity field of a vortex and the magnetic field of a straight current carrying wire are both circular. We have redrawn Figure (29-10) from the last chapter, showing the circular magnetic field around a wire.

The formula for the strength of the magnetic field in Figure (29-10) is

$$B \times 2\pi r = \mu_0 i \qquad (28\text{-}18)$$

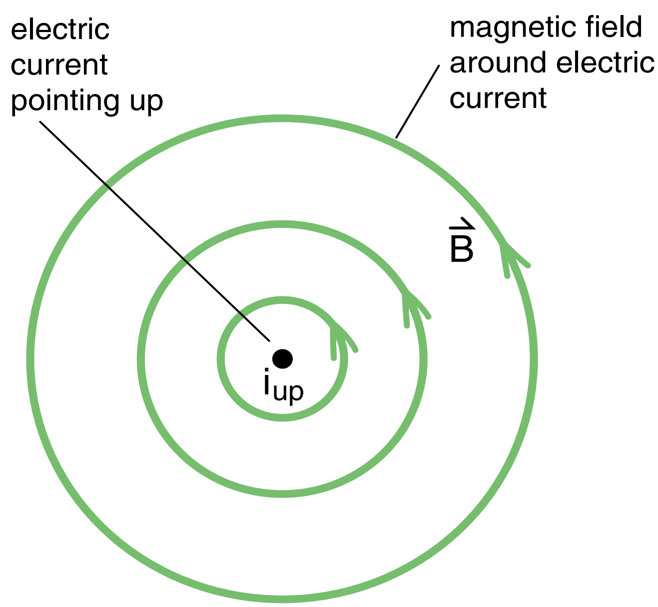
a result we derived back in Equation 28-18. This should be compared with the formula for the strength of the electric field in Figure (13)

$$E \times 2\pi r = -\frac{d\Phi_B}{dt} \qquad (16)$$



$$E \times 2\pi r = -\frac{d\Phi_B}{dt}$$

Figure 13
Circular electric field around a changing magnetic flux.



$$B \times 2\pi r = \mu_0 i$$

Figure 29-10
Circular magnetic field around an electric current.

In our discussion of Ampere's law, we called $\mu_0 i$ the "source" of the circular magnetic field. By analogy, we should think of the rate of change of magnetic flux, $-d\Phi_B/dt$, as the "source" of the circular electric field.

In Chapter 29, we generalized Ampere's law by replacing $B * 2\pi r$ by the line integral $\oint \vec{B}\cdot d\vec{\ell}$ along a closed path around the wire. The result was

$$\boxed{\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i} \qquad \textit{Ampere's law for magnetic fields} \qquad (29\text{-}18)$$

where the line integral can be carried out along any closed path surrounding the wire. Because of close analogy between the structure and magnitude of the magnetic field in Figure (29-10) and the electric field in Figure (13), we expect that the more general formula for the electric field produced by a changing magnetic flux is

$$\boxed{\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}} \qquad \textit{Faraday's law for electric fields} \qquad (17)$$

Equation 17 is the most general form of Faraday's law. It says that the line integral of the electric field around any closed path is equal to (minus) the rate of change of magnetic flux through the path.

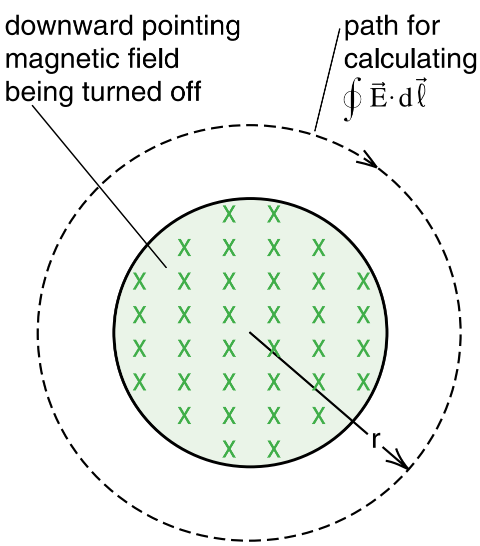


Figure 13a
Using Faraday's law to calculate $\vec{E}$.

USING FARADAY'S LAW

Up until now we have been looking for arguments leading up to Faraday's law. Let us now reverse the procedure, treating Equation 17 as a basic law for electric fields, and see what the consequences are.

Electric Field of an Electromagnet

As a beginning exercise in the use of Faraday's law, let us use Equation (17) to calculate the electric field of the electromagnet in Figure (13). We first argue that because of the circular symmetry, the electric field should travel in circles around the decreasing magnetic field. Thus we choose a circular path, shown in Figure (13a), along which we will calculate $\oint \vec{E}\cdot d\vec{\ell}$. Then using the assumption (because of circular symmetry) that $\vec{E}$ is parallel to $d\vec{\ell}$ and has a constant magnitude all the way around the circular path, we can write

$$\oint \vec{E}\cdot d\vec{\ell} = \oint E\,d\ell = E\oint d\ell = E\,2\pi r \qquad (18)$$

Using this result in Equation (17) gives

$$\oint \vec{E}\cdot d\vec{\ell} = E\,2\pi r = -\frac{d\Phi_B}{dt} \qquad (19)$$

which is the result we had in Equation (16).

Right Hand Rule for Faraday's Law

We can get the correct direction for $\vec{E}$ with the following right hand rule. Point the thumb of your right hand in the direction of the magnetic field. If the magnetic flux is decreasing (if $-d\Phi_B/dt$ is positive), then the fingers of your right hand curl in the direction of $\vec{E}$. If the magnetic flux is increasing, then $\vec{E}$ points the other way. ***Please practice this right hand rule on Figures (13a), (9c), and (15).***

Electric Field of Static Charges

If all we have around are static electric charges, then there are no magnetic fields, no magnetic flux, and no changing magnetic flux. For this special case, $d\Phi_B/dt = 0$ and Faraday's law gives

$$\oint \vec{E}\cdot d\vec{\ell} = 0 \quad \left\{ \begin{array}{l} \text{for electric fields} \\ \text{produced by} \\ \text{static charges} \end{array} \right. \tag{20}$$

When the line integral of a force is zero around any closed path, we say that the force is ***conservative***. (See Equation 29-12.) Thus we see that if we have only static electric charge (or constant magnetic fields), the electric field is a conservative field.

In contrast, if we have changing magnetic fields, if $d\Phi_B/dt$ is not zero, the electric field is not conservative. This can lead to some rather interesting results which we will see in our discussion of a device called the *betatron*.

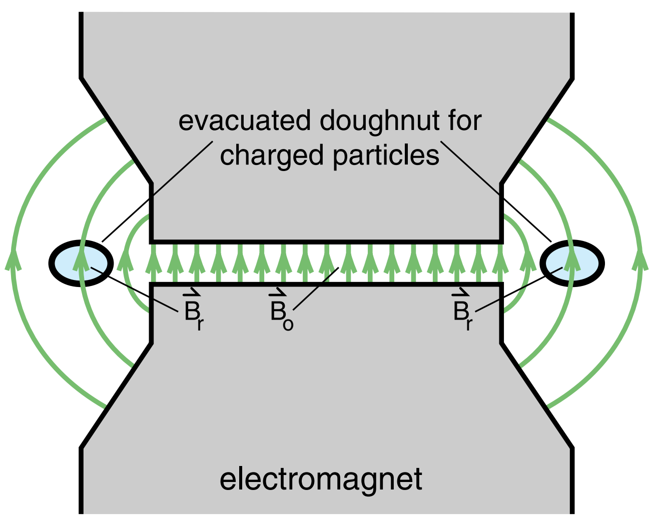


Figure 14a
Cross-sectional view of a betatron, showing the central field $\vec{B}_0$ and the field $\vec{B}_r$ out at the evacuated doughnut. The relative strength of $\vec{B}_0$ and $\vec{B}_r$ can be adjusted by changing the shape of the electromagnet pole pieces.

THE BETATRON

As we have mentioned before, when you encounter a new and strange equation like Faraday's law, it is essential to have an example that you know inside out that illustrates the equation. This transforms the equation from a collection of symbols into a set of instructions for solving problems and making predictions. One of the best examples to learn for the early form of Faraday's law, Equation (15a), was the air cart speed detector experiment shown in Figure (7). (You should have done Exercise 2 analyzing the experiment using Equation (15a).

The most direct example illustrating Faraday's law for electric fields, Equation (17), is the particle accelerator called the betatron. This device was used in the 1950s for study of elementary particles, and later for creating electron beams for medical research.

A cross-sectional view of the betatron is shown in Figure (14a). The device consists of a large electromagnet with a circular evacuated doughnut shaped chamber for the electrons. The circular shape of the electromagnet and the evacuated chamber are more clearly seen in the top view, Figure (14b). In that view we show the strong upward directed magnetic field $\vec{B}_0$ in the gap and the weaker upward directed magnetic field out at the evacuated doughnut.

The outer magnetic field B_r is required to keep the electrons moving along a circular orbit inside the evacuated chamber. This field exerts a force $\vec{F}_B = (-e)\,\vec{v}\times\vec{B}_r$ that points toward the center of the circle and has a magnitude mv^2/r in order to produce the required radial acceleration. Thus B_r is given by

$$B_r = \frac{mv}{er}$$

which is our familiar formula for electrons moving along a circular path in a magnetic field. *(As a quick review, derive the above equation.)*

Since a magnetic field does no work we need some means of accelerating the electrons. In a synchrotron, shown in Figure (28-27), a cavity which produces an electric accelerating field is inserted into the electron's path. As an electron gains energy and momentum (mv) each time it goes through the cavity, the magnetic field

B was increased so that the electron's orbital radius $r = mv/eB$ remains constant. (The synchronizing of B with the momentum mv leads to the name synchrotron.)

In the betatron of Figure (14), we have a magnetic field B_r to keep the electrons in a circular orbit, and as the electrons are accelerated, B_r is increased to keep the electrons in an orbit of constant radius r. But what accelerates the electrons? There is no cavity as in a synchrotron.

Suppose that both B_0 and B_r are increased simultaneously. In the design shown in Figure (14a), $\vec{B}_0$ and $\vec{B}_r$ are produced by the same electromagnet, so that we can increase both together by turning up the electromagnet. If the strong central field $\vec{B}_0$ is increased, we have a large change in the magnetic flux through the electron orbit, and therefore by Faraday's law $\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$ we must have a circular electric field around the flux as shown in Figure (15), just as in Figure (13). This electric field is exactly parallel to the orbit of the electrons and accelerates them continuously as they go around.

What is elegant about the application of Faraday's law to the electrons in the betatron, is that $\oint \vec{E} \cdot d\vec{\ell}$, which has the dimensions of voltage, is the ***voltage gained by an electron going once around the circular orbit. The energy gained is just this voltage in electron volts***

$$\left.\begin{array}{l}\text{energy gained}\\ \text{(in eV) by electron}\\ \text{going around once}\end{array}\right\} = \oint \vec{E} \cdot d\vec{\ell} \tag{21}$$

This voltage is then related to $d\Phi_B/dt$ by Faraday's law.

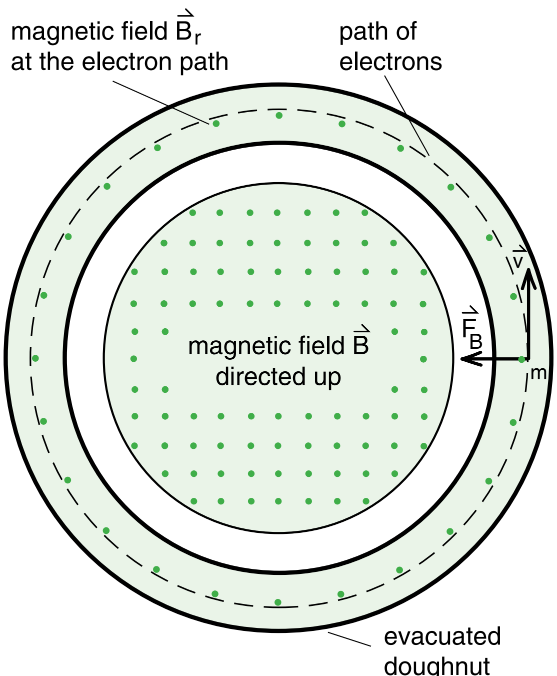


Figure 14b
Top view of the betatron showing the evacuated doughnut, the path of the electrons, and the magnetic fields $\vec{B}_0$ in the center and $\vec{B}_r$ out at the electron path. In order to keep the electrons moving on a circular path inside the doughnut, the magnetic force $\vec{F}_B = (-e)\vec{v} \times \vec{B}_r$ must have a magnitude $\vec{F}_B = mv^2/r$ where r is the radius of the evacuated doughnut.

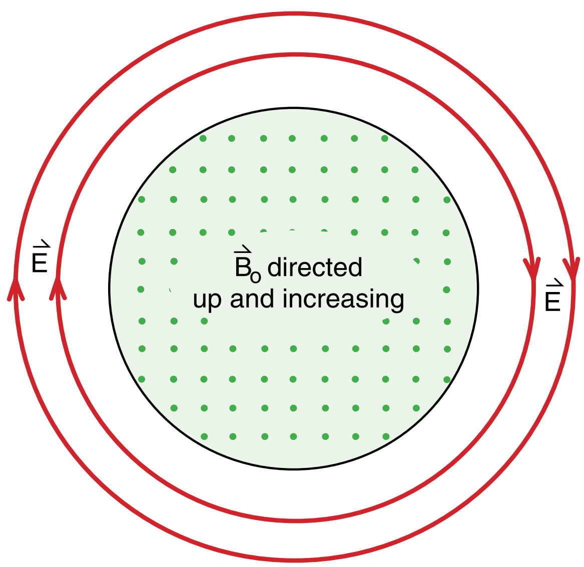


Figure 15
When the strong central field $\vec{B}_0$ in the betatron is rapidly increased, it produces a circular electric field that is used to accelerate the electrons. The electric field $\vec{E}$ is related to the flux Φ_B of the central field $\vec{B}_0$ by Faraday's law $\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$.

Let us consider an explicit example to get a feeling for the kind of numbers involved. In the 100 MeV betatron built by General Electric, the electron orbital radius is 84 cm, and the magnetic field B_0 is cycled from 0 to .8 tesla in about 4 milliseconds. (The field B_0 is then dropped back to 0 and a new batch of electrons are accelerated. The cycle is repeated 60 times a second.)

The maximum flux Φ_m through the orbit is

$$\Phi_m = \left(B_0\right)_{max}\pi r^2 = .8\ \text{tesla} \times \pi \times (.84\text{m})^2$$

$$\Phi_m = 1.8\ \text{tesla}\ \text{m}^2$$

If this amount of flux is created in 4 milliseconds, then the average value of the rate of change of magnetic flux Φ_B is

$$\frac{d\Phi_B}{dt} = \frac{\Phi_m}{.004\ \text{sec}} = \frac{1.8}{.004} = 450\ \text{volts}$$

Thus each electron gains 450 electron volts of kinetic energy each time it goes once around its orbit.

Exercise 3

(a) How many times must the electron go around to reach its final voltage of 100 MeV advertised by the manufacturer?

(b) For a short while, until the electron's kinetic energy gets up to about the electron's rest energy m_0c^2, the electron is traveling at speeds noticeably less than c. After that the electron's speed remains very close to c. How many orbits does the electron have to make before its kinetic energy equals its rest energy? What fraction of the total is this?

(c) How long does it take the electron to go from the point that its kinetic energy equals its rest energy, up to the maximum of 100 MeV? Does this time fit within the 4 milliseconds that the magnetic flux is being increased?

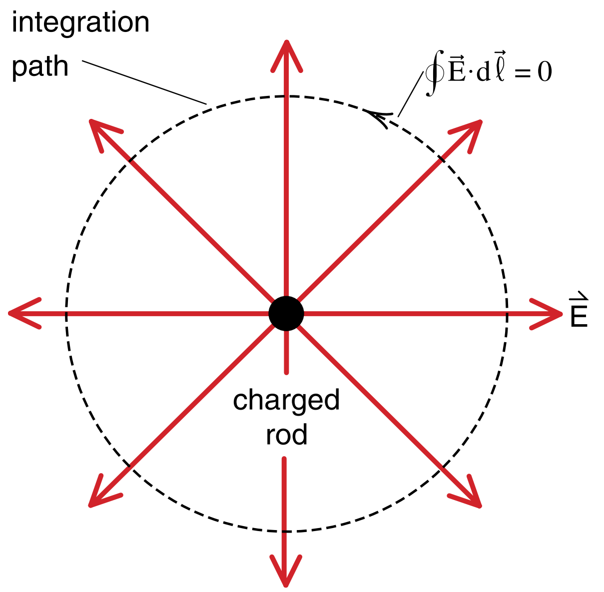
TWO KINDS OF FIELDS

At the beginning of the chapter we showed that the line integral $\oint \vec{E} \cdot d\vec{\ell}$ around a closed path was zero for any electric field produced by static charges. Now we see that the line integral is not zero for the electric field produced by a changing magnetic flux. Instead it is given by Faraday's law $\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$. These results are shown schematically in Figure (16) where we are looking at the electric field of a charged rod in (16a) and a betatron in (16b).

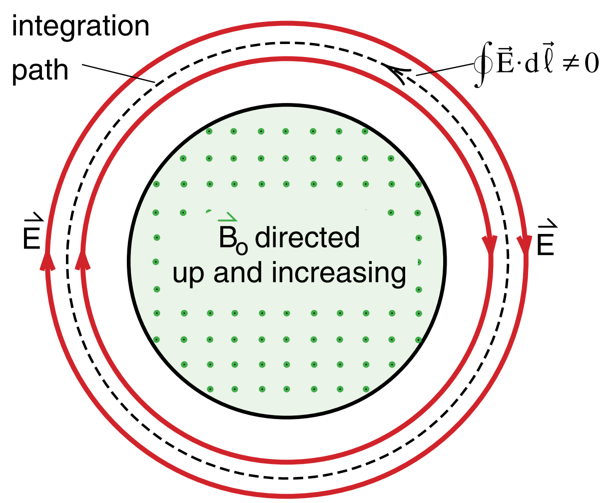
In Figure (17), we have sketched a wire loop with a voltmeter, the arrangement we used in Figure (12) to measure the $\oint \vec{E} \cdot d\vec{\ell}$. We will call this device an "$\oint \vec{E} \cdot d\vec{\ell}$ ***meter***". If you put the $\oint \vec{E} \cdot d\vec{\ell}$ meter over the changing magnetic flux in Figure (16b), the voltmeter will show a reading of magnitude $V = d\Phi_B/dt$. If we put the $\oint \vec{E} \cdot d\vec{\ell}$ meter over the charged rod in Figure (16a), the meter reads $V = 0$. Thus we have a simple physical device, our $\oint \vec{E} \cdot d\vec{\ell}$ meter, which can distinguish the radial field in Figure (16a) from the circular field in Figure (16b). In fact it can distinguish the circular field in (16b) from any electric field $\vec{E}$ whatsoever that we can construct from static charges. Our $\oint \vec{E} \cdot d\vec{\ell}$ meter allows us to separate all electric fields into two kinds, those like the one in (16b) that can give a ***non zero reading***, and those, produced by static charges, which give a ***zero reading***.

Fields which register on our $\oint \vec{E} \cdot d\vec{\ell}$ meter generally close on themselves like the circular fields in (16b). Since these fields do not appear to have sources, they are called sourceless or "***solenoidal***" fields. An $\oint \vec{E} \cdot d\vec{\ell}$ meter is the kind of device we need to detect solenoidal fields.

The conservative fields produced by static charges never close on themselves. They always start on positive charge, end on negative charge, or come from or go to infinity. These fields diverge from point charges and thus are sometimes called ***"divergent"*** fields. Our $\oint \vec{E} \cdot d\vec{\ell}$ meter does not work on the divergent fields because we always get a zero reading.



(a) Electric field of a static charge distribution has the property $\oint \vec{E} \cdot d\vec{\ell} = 0$



(b) Electric field produced by a changing magnetic flux has $\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$

Figure 16
Two kinds of electric field. Only the field produced by the changing magnetic flux has a non zero line integral.

Although the $\oint \vec{E} \cdot d\vec{\ell}$ meter does not work on divergent fields, Gauss' law with the surface integral does. In a number of examples we used Gauss' law

$$\int_{\substack{\text{closed}\\\text{surface}}} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad (29\text{-}5)$$

to calculate the electric field of static charges. We are seeing now that we ***use a surface integral to measure divergent fields, and a line integral to measure solenoidal fields***. There are two kinds of electric fields, and we have two kinds of integrals to detect them.

It turns out to be a general mathematical theorem that any vector field can be separated into a purely divergent part and a purely solenoidal part. The field can be uniquely specified if we have both an equation involving a Gauss' law type surface integral to tell us the divergent part, and an equation involving a Faraday's law type line integral to tell us the solenoidal part.

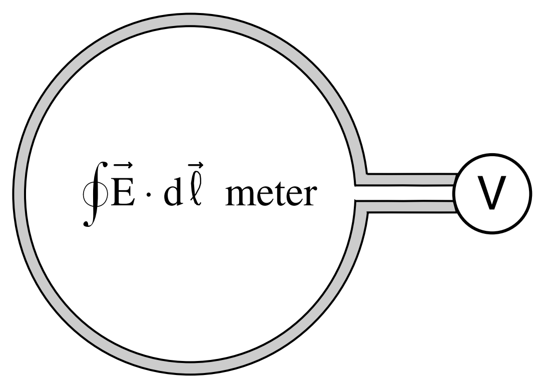


Figure 17
Wire loop and a volt meter can be used directly to measure $\oint \vec{E} \cdot d\vec{\ell}$ around the loop. We like to call this apparatus an $\oint \vec{E} \cdot d\vec{\ell}$ meter.

Exercise 4

a) Maxwell's equations are a set of equations that completely define the behavior of electric fields $\vec{E}$ and magnetic fields $\vec{B}$. One of Maxwell's equations is Faraday's law

$$\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$$

which gives the line integral for the electric field. *How many Maxwell equations are there?* (How many equations will it take to completely define both $\vec{E}$ and $\vec{B}$?)

b) Are any of the other equations for electric and magnetic fields we have discussed earlier, candidates to be one of Maxwell's equations?

c) At least one of Maxwell's equations is missing – we have not discussed it. Can you guess what the equation is and write it down? Explain what you can about your guess.

d) Back in our early discussion of velocity fields and Gauss' law, we said that a *point source* for the velocity field of an incompressible fluid like water, was a small "magic" sphere in which water molecules were created. Suppose we do not believe in magic and assume that for real water there is no way that water molecules can be created or destroyed. Write down an integral equation for real water that expresses the fact that the $\vec{v}_{\text{real water}}$ has no sources (that create water molecules) or sinks (that destroy them).

Do the best you can on these exercises now. Keep a record of your work, and see how well you did when we discuss the answers later in chapter 32.

Note on our $\oint \vec{E} \cdot d\vec{\ell}$ meter

Back in Figure (17) we used a wire loop and a voltmeter as an $\oint \vec{E} \cdot d\vec{\ell}$ meter. I.e., we are saying that the voltage reading V on the voltmeter gives us the integral of $\vec{E}$ around the closed path defined by the wire loop. This is strictly true for a loop at rest, where the conduction electrons experience no magnetic force and all forces creating the electric pressure are caused by the electric field $\vec{E}$.

Earlier, in Figure (9), we had two views of an $\oint \vec{E} \cdot d\vec{\ell}$ meter. In the bottom view, (9b) the loop is at rest and the voltage must be caused by an electric force. The moving magnetic field must have an electric field associated with it. But in Figure (9a) where the magnet is at rest, there is no electric field and the voltage reading is caused by the magnetic force on the conduction electrons in the moving wire. Strictly speaking, in Figure (9a) the wire loop and voltmeter are measuring a pressure caused by magnetic forces and not an $\oint \vec{E} \cdot d\vec{\ell}$. The wire loop must be at rest, ***the path for our line integral cannot move****, if we are measuring $\oint \vec{E} \cdot d\vec{\ell}$.*

In practice, however, it makes little difference whether we move the magnet or the loop, because the principle of relativity requires that we get the same voltage V.

APPLICATIONS OF FARADAY'S LAW

The last few sections have been somewhat heavy on theory. To end this chapter on a more practical note, we will consider some simple applications of Faraday's law, one that has immense practical applications and another that we can use in the laboratory. First we will discuss the AC voltage generator which is used by most power stations throughout the world. We will also describe a field mapping experiment in which we use our $\oint \vec{E} \cdot d\vec{\ell}$ meter to map the magnetic field of a pair of Helmholtz coils. In the next chapter Faraday's law is used to explain the operation of transformers and inductors that are common circuit elements in radio and television sets.

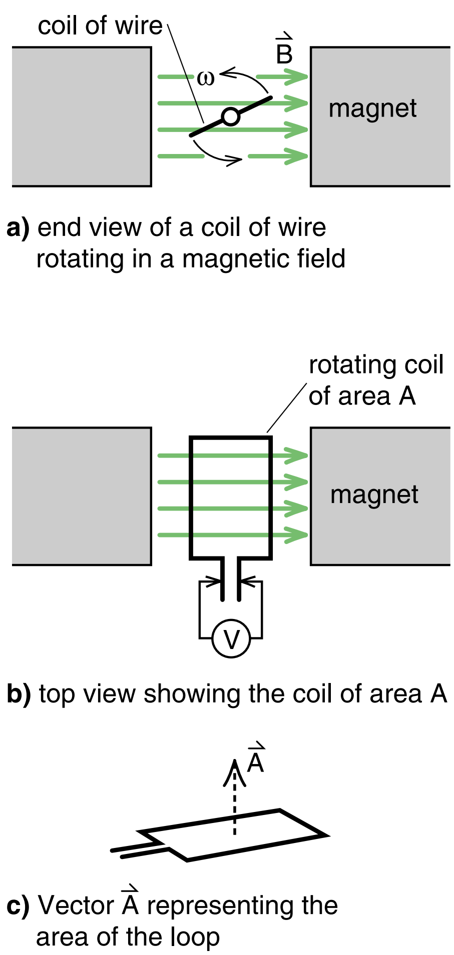


Figure 18
An electric generator consists of a coil of wire rotating in a magnetic field.

The AC Voltage Generator

In Figure (18) we have inserted a wire loop of area $\vec{A}$ in the magnetic field $\vec{B}$ of a magnet. We then rotate the coil at a frequency ω about an axis of the coil as shown. We also attach a voltmeter to the coil, using sliding contacts so that the voltmeter leads do not twist as the coil spins.

As shown in Figure (19), as the loop turns, the magnetic flux changes sinusoidally from a maximum positive flux in (19a) to zero flux in (c) to a maximum negative flux in (d) to zero in (e). In (18c), we have shown the vector $\vec{A}$ representing the area of the coil ($\vec{A}$ points

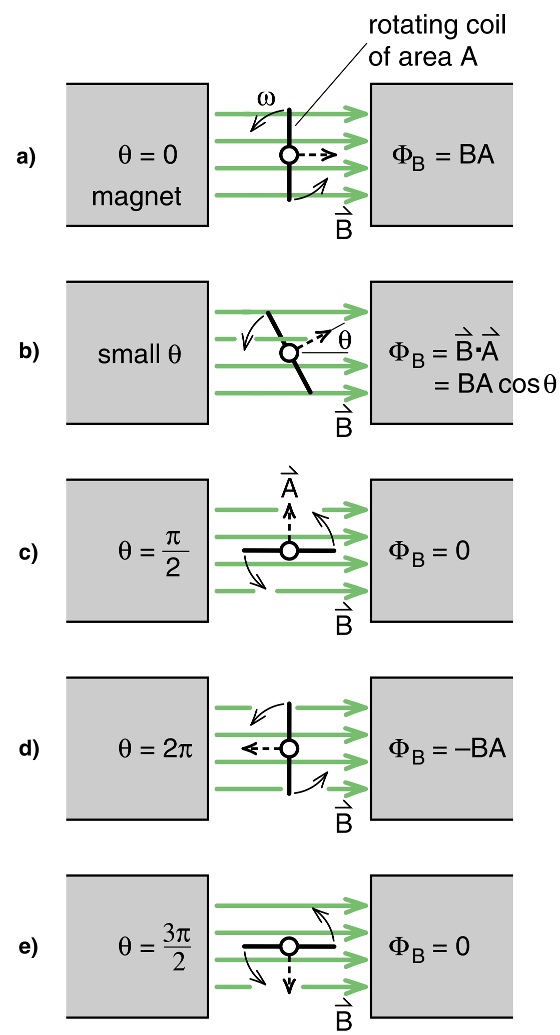


Figure 19
The changing magnetic flux through the rotating loop. The general formula for Φ_B is $BA\cos\theta$ where θ is the angle shown in (b), between the magnetic field and the normal to the loop. If the coil is rotating uniformly, then $\theta = \omega t$, and $\Phi_B = BA\cos\omega t$

perpendicular to the plane of the coil) and we can use our usual formula for magnetic flux to get

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta \tag{22}$$

If the coil is rotating at a constant angular velocity ω, then $\theta = \omega t$ and we have

$$\Phi_B = BA\cos\omega t \tag{23}$$

Differentiating Equation (23) with respect to time gives

$$\frac{d\Phi_B}{dt} = -\omega BA\sin\omega t \tag{24}$$

Finally we use Faraday's law in the form

$$V = -\frac{d\Phi_B}{dt} \tag{15}$$

to predict that the voltage V on the voltmeter will be

$$V = \omega BA\sin\omega t \tag{25}$$

If we use a coil with N turns, we get a voltage N times as great, or

$$V = \omega NBA\sin\omega t = V_0\sin\omega t \tag{26}$$

where V_0 is the amplitude of the sine wave as shown in Figure (20). Equation (26) shows that by rotating a coil in a magnetic field, we get an alternating or "AC" voltage. Power stations use this same principle to generate AC voltages.

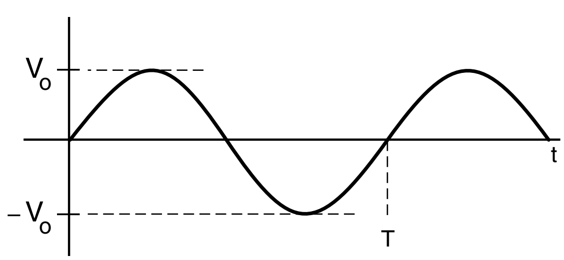


Figure 20
Amplitude and period of a sine wave.

Equation 26 predicts that the voltage amplitude V_0 produced by an N turn coil of area A rotating in a magnetic field B is

$$V_0 = \omega NBA \tag{27}$$

where the angular frequency ω radians per second is related to the frequency f cycles per second and the period T seconds per cycle by

$$\omega\frac{\text{rad}}{\text{sec}} = 2\pi\frac{\text{rad}}{\text{cycle}}\times f\frac{\text{cycle}}{\text{sec}} = 2\pi\frac{\text{rad}}{\text{cycle}}\times\frac{1}{T\frac{\text{sec}}{\text{cycle}}}$$

Exercise 5

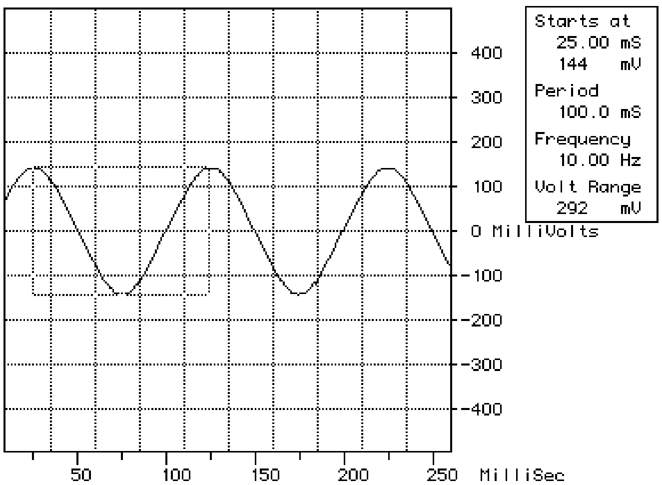
Suppose that you have a magnetic field B = 1 tesla, and you rotate the coil at 60 revolutions (cycles) per second. Design a generator that will produce a sine wave voltage whose amplitude is 120 volts.

Exercise 6

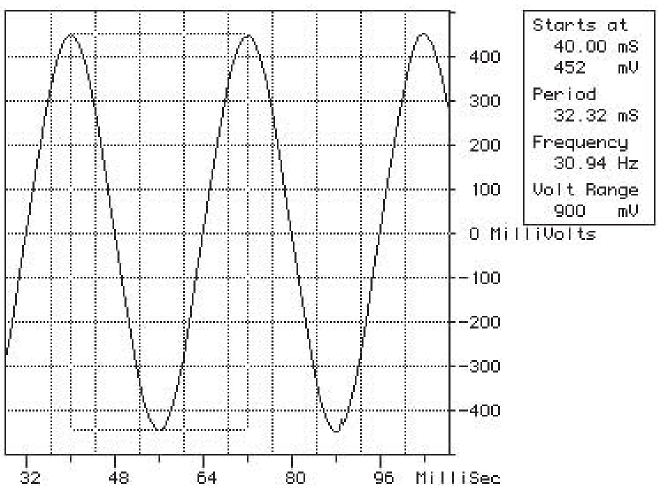
Figures (21a,b) show the voltage produced by a coil of wire rotating in a uniform magnetic field of a fairly large electromagnet. (The setup is similar to that shown in Figures 18 and 19.) The coil was square, 4 cm on a side, and had 10 turns. To go from the results shown in Figure (21a) to those shown in Figure (21b), we increased the rotational speed of the motor turning the coil. In both diagrams, we have selected one cycle of the output wave, and see that the frequency has increased from 10 cycles per second to nearly 31 cycles per second.

a) Explain why the amplitude of the voltage signal increased in going from Figure (21a) to (21b). Is the increase what you expected?

b) Calculate the strength of the magnetic field of the electromagnet used. Do you get the same answer using Figure (21a) and using Figure (21b)?



a)



b)

Figure 21
Voltage output from a coil rotating in a uniform magnetic field. The coil was 4 cm on a side, and had 10 turns. In each figure we have selected one cycle of the output wave, and see that the frequency of rotation increased from 10 cycles per second in a) to nearly 31 cycles per second in b).

Gaussmeter

Exercise 6 demonstrates one way to measure the strength of the magnetic field of a magnet. By spinning a coil in a magnetic field, we produce a voltage amplitude given by Equation 27 as $V_0 = \omega NBA$. Thus by measuring V_0, ω, N, and A, we can solve for the magnetic field B.

A device designed to measure magnetic fields is called a ***gaussmeter***. A commercial gaussmeter, used in our plasma physics lab, had a small coil mounted in the tip of a metal tube as shown in Figure (22). A small motor also in the tube spun the coil at high speed, and the amplitude V_0 of the coil voltage was displayed on a meter. The meter could have been calibrated using Equation (27), but more likely was calibrated by inserting the spinning coil into a known magnetic field.

In an attempt to measure the magnetic field in the Helmholtz coils used for our electron gun experiments, students have also built rotating coil gaussmeters. Despite excellent workmanship, the results were uniformly poor. The electrical noise generated by the sliding contacts and the motor swamped the desired signal except when B was strong. This approach turned out not to be the best way to measure $\vec{B}$ in the Helmholtz coils.

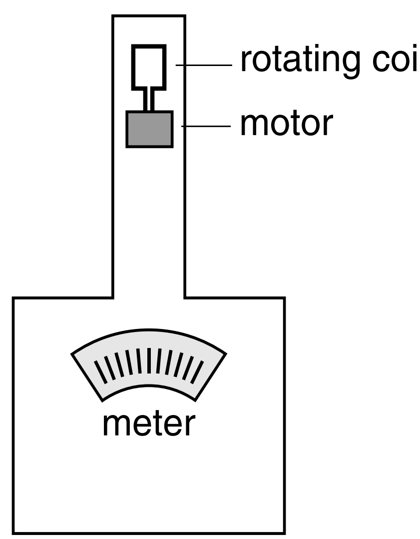


Figure 22
A commercial gauss meter, which measures the strength of a magnetic field, has a motor and a rotating coil like that shown in Figure 18. The amplitude V_0 of the voltage signal is displayed on a meter that is calibrated in gauss.

A Field Mapping Experiment

To measure the magnetic field in the Helmholtz coils, it is far easier to "rotate the field" than the detector loop. That is, use an alternating current in the Helmholtz coils, and you will get an alternating magnetic field in the form

$$B = B_0 \sin \omega t \tag{28}$$

where w is the frequency of the AC current in the coils. Simply place a stationary detector loop in the magnetic field as shown in Figure (23) and the magnetic flux through the detector loop will be

$$\Phi_B = \vec{B}\cdot\vec{A} = \vec{B}_0\cdot\vec{A}\sin\omega t \tag{29}$$

where $\vec{A}$ is the area of the detector loop. By Faraday's law, the voltage in the voltmeter or oscilloscope attached to the detector loop is given by

$$V = -\frac{d\Phi_B}{dt} = -\left(\omega\vec{B}_0\cdot\vec{A}\right)\cos\omega t \tag{30}$$

If our detector loop has N turns of wire, then the voltage will be N times as great, and the amplitude V_0 we see on the oscilloscope screen will be

$$\boxed{V_0 = N\omega\left(\vec{B}_0\cdot\vec{A}\right)} \tag{31}$$

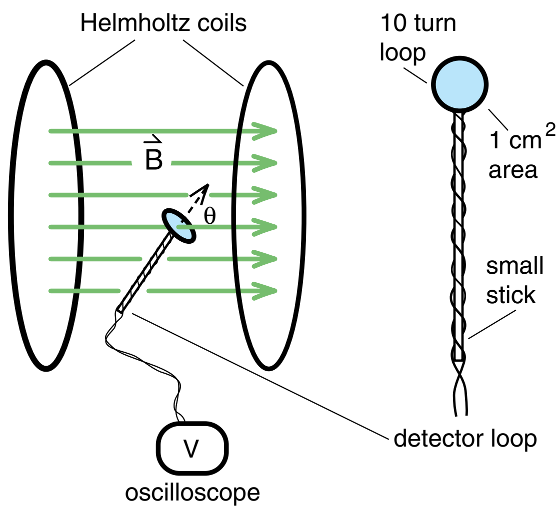


Figure 23
If you use an alternating current in the Helmholtz coils, then B has an alternating amplitude $B = B_0\cos(\omega t)$. You can then easily map this field with the detector loop shown above. If you orient the loop so that the signal on the oscilloscope is a maximum, then you know that $\vec{B}$ is perpendicular to the detector loop and has a magnitude given by
$V = V_0\sin(\omega t) = d\Phi_B/dt = d/dt\left(NAB\cos(\omega t)\right)$.

This is essentially the same formula we had for the rotating coil gaussmeter, Equation (27). The difference is that by "rotating the field" rather than the coil, we avoid sliding contacts, motors, electrical noise, and can make very precise measurements.

A feature of Equation (31) that we did not have when we rotated the coil is the dot product $\vec{B}_0\cdot\vec{A}$. When the detector coil is aligned so that its area vector $\vec{A}$ (which is perpendicular to the plane of the detector coil) is parallel to $\vec{B}_0$, the dot product $\vec{B}_0\cdot\vec{A}$ is a maximum. Thus we not only measure the magnitude of $\vec{B}_0$, we also get the direction by reorienting the detector coil until the V_0 is a maximum.

As a result, a small coil attached to an oscilloscope, which is our $\oint\vec{E}\cdot d\vec{\ell}$ meter, can be used to accurately map the magnitude and direction of the magnetic field of the Helmholtz coils, or of any coil of wire. Unlike our earlier electric field mapping experiments, there are no mysteries or unknown constants. Faraday's law, through Equation (31), gives us a precise relation between the observed voltage and the magnetic field. The experimental setup is seen in Figure (24).

Still another way to measure magnetic fields is illustrated in Exercise 7.

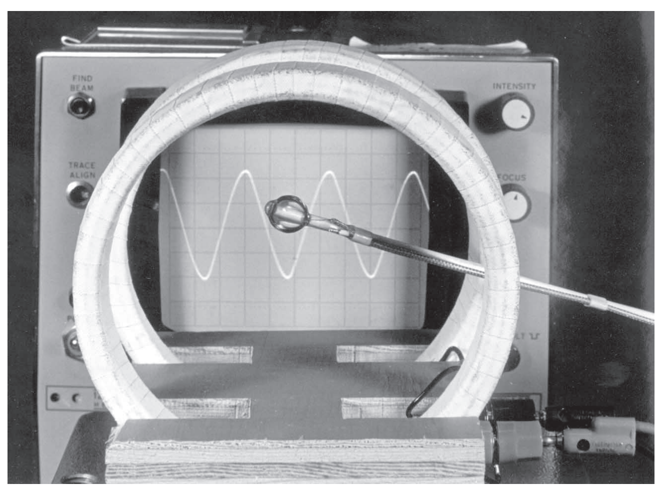


Figure 24
Experimental setup for the magnetic field mapping experiment. A 60 cycle AC current is running through the Helmholtz coils, producing an alternating magnetic flux through the 10 turn search coil. The resulting induced voltage is seen on the oscilloscope screen.

Exercise 7

The point of this experiment is to determine the strength of the magnetic field produced by the small magnets that sat on the angle iron bars in the velocity detector apparatus. We placed a short piece of wood between two magnets so that there was a small gap between the ends as seen in the actual size computer scan of Figure (25). The pair of magnets were then suspended over the air track as shown in Figure (26).

On top of the air cart we mounted a **single turn** coil. When the air cart passes under the magnets, the single turn coil passes through the lower gap between the magnets as shown. The dimensions of the single turn coil are shown in Figure 27. We also show the dimensions and location of the lower end of one of the magnets at a time when the coil has passed part way through the gap. You can see that, at this point, all the magnetic flux across the lower gap is passing completely through the single turn coil. Figure 28 is a recording of the induced voltage in the single turn coil as the coil passes completely through the gap. The left hand blip was produced when the coil entered the gap, and the right hand blip when the coil left the gap. The air track was horizontal, so that the speed of the air cart was constant as the coil moved through the gap. Determine the strength of the magnetic field B in the gap. Show and explain your work.

Figure 25
Two C Magnets with wood spacer.

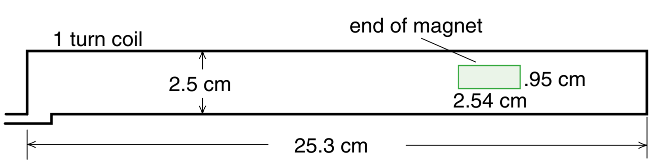


Figure 27
Dimensions of the single turn coil. We also show the dimensions of the end of the magnets through which the coil is passing.

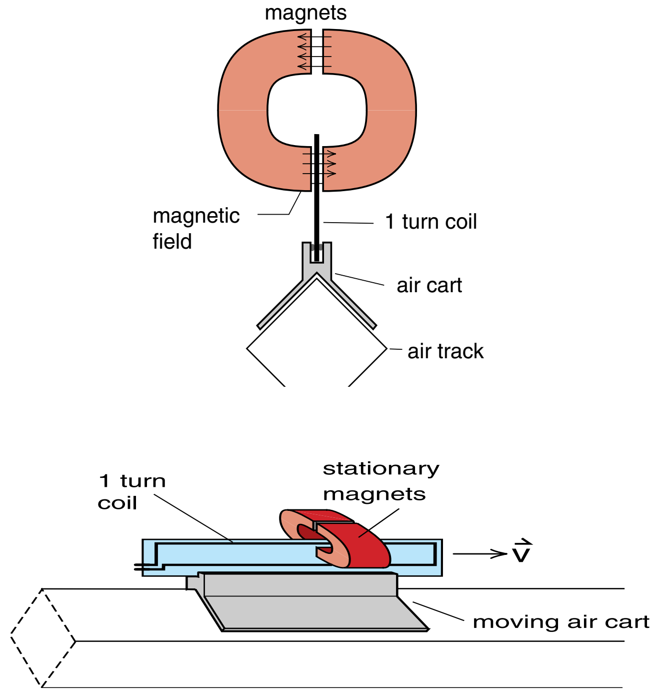


Figure 26
A single turn coil, mounted on an air cart, moves through the lower gap between the magnets.

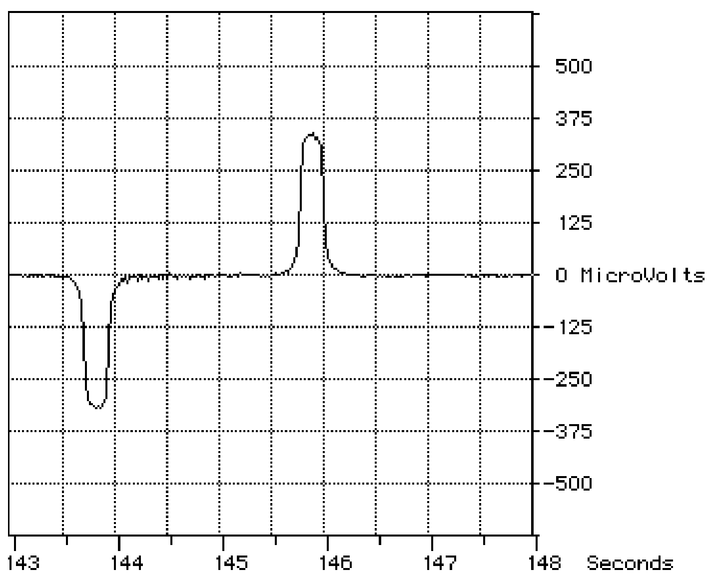


Figure 28
Voltage induced in the single turn coil.

Exercise 8

As shown in Figure (29), we started with a solenoid with 219 turns wrapped in a 1" diameter plastic tube. The coil is 45.4 cm long. The current going through the coil first goes through a .1Ω resistor. By measuring the voltage V_1 across that resistor, we can determine the current through the solenoid. V_1 is shown as the lower curve in Figure (30).

a) Using V_1 from Figure 30, calculate the magnitude B of the magnetic field in the solenoid.

We then wound 150 turns of wire around the center section of the solenoid, as indicated in Figure (29). You can see that the entire flux Φ_1 of the Magnetic field of the solenoid, goes through all the turns of the outer coil.

b) Use this fact to predict the voltage V_2 across the outer coil, and then compare your prediction with the experimental V_2 shown in the upper curve of Figure (30).

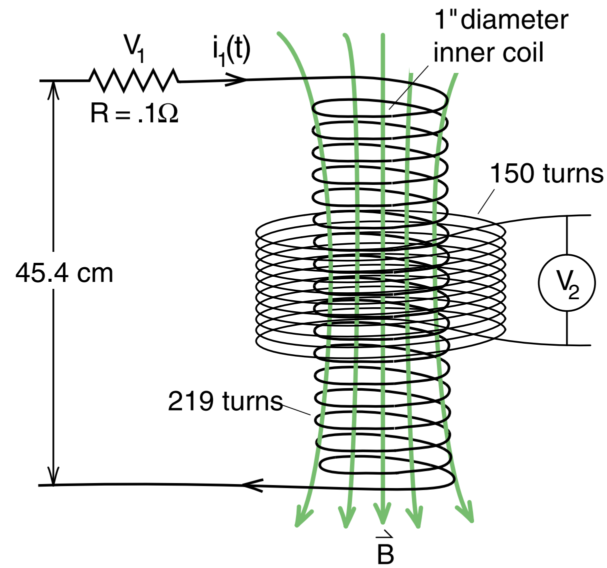


Figure 29
The inner (primary) coil 1 is 45.4 cm long, has 219 turns and is wound on a 2.54 cm (1") diameter tube. The outer (secondary) coil consists of 150 turns wound tightly around the center section of the primary coil. The current through the primary coil goes through a .1Ω resistor, and the voltage V_1 is measured across that resistor. V_2 is the voltage induced in the secondary coil.

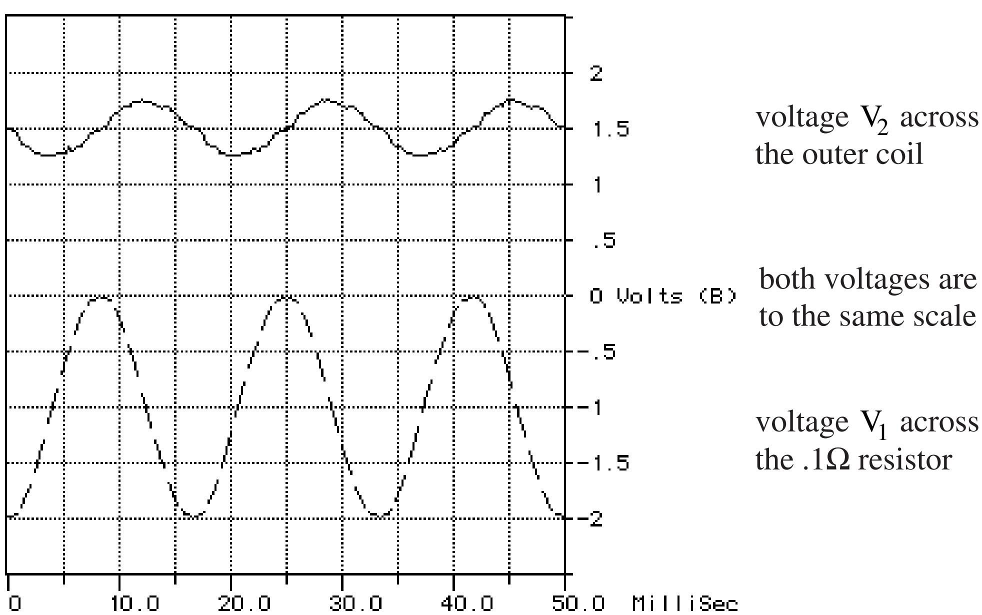


Figure 30
The voltage V_1 across the .1Ω resistor measures the current in the primary (219 turn) coil. V_2 is the voltage induces in the secondary (outer 150 turn) coil.

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